Math-GAMES Guidebook

GAMES AND MATHEMATICS IN EDUCATION FOR ADULTS
COMPENDIUMS, GUIDELINES AND COURSES
FOR NUMERACY LEARNING METHODS BASED ON GAMES

ENGLISH

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www.math-games.eu

www.math-games.eu

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Preliminary Remarks

Contribution for the Preparation of this Guidebook

The Guidebook is the outcome of the collaborative work of all the Partners for the development of the European Erasmus+ Math-GAMES Project, namely the following:

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The complete output of the project Math GAMES consists of the here present **Compendium** and a **Guidebook**, a **Teacher Training Course and Seminar** and an **Evaluation Report**, mostly translated into nine European languages. You can download all from the website www.math-games.eu as an active PDF-version, where you can “click” on the content to go directly to the page or “click” on the links to open pages in any browser.

You can find most of the games of this project on the **YouTube Math-GAMES-Channel**:
https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

This book is also available as a print-version from Verlag Ludwig Schulbuch: www.ludwig-schulbuch.de

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Introduction to the Math-Games Project

Using games to develop numeracy

Games can help learners to practice matching, counting and computational skills such as doubling, addition, subtraction and tables. Some games combine these skills with strategy, and this can help learners to develop problem-solving skills. Children's boards' games or dominoes can be used in family numeracy sessions. Adult games include bingo, dominoes, card games, strategy games such as backgammon, and traditional African games such as Oware and Ayo, which are now available commercially.

Statement from the "Adult Numeracy Core Curriculum", London, 2001

More than 13% of all people in Europe cannot read, write or count. Therefore it is the declared goal of the European Union to remedy this situation and to reduce the number of poorly trained people. The project Math-GAMES has been developed within this environment; its title says it all: ***"Math Games - Games and Mathematics in Education for Adults - Compendium, Guidelines and Courses for Numeracy Learning Methods Based on Games (Mathematical Literacy)".*** In the project books and hand-outs will be created, such as the present compendium together with the guidebook, which should give an answer in nine languages to the following questions:

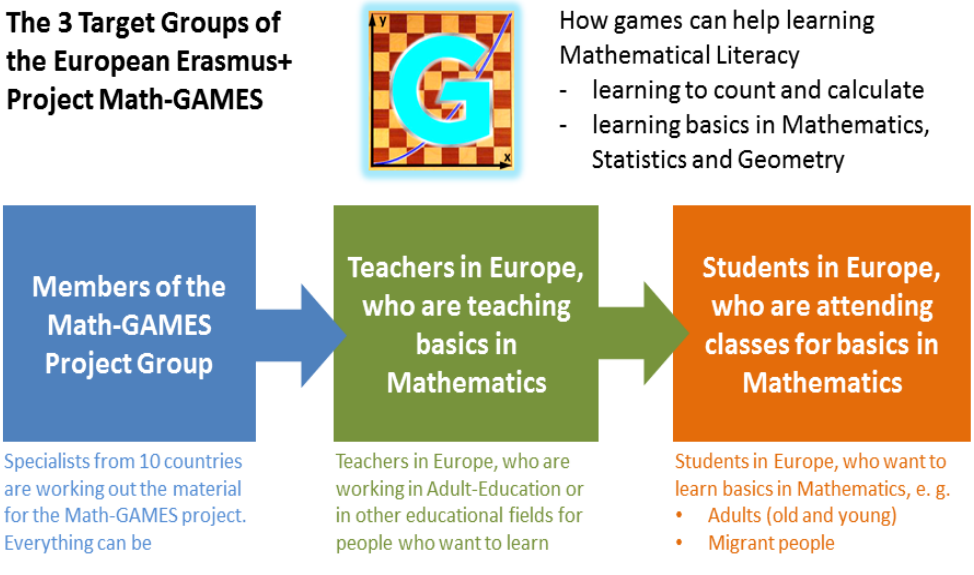
1. How can we reduce the number of under skilled adults to promote social integration and participation into our society?
2. How can we increase incentives for adult training by using games?
3. How can we offer tailored learning opportunities to individual learners by using games?
4. How can we provide information on accessing to the services of adult learning?
5. How can we save traditional and famous games in different countries from being lost?

The Math-GAMES Project will give the following answers:

To 1: We can reduce the number of under skilled adults to promote social integration and participation into our society by giving them the opportunity to learn what they need in their jobs or in other fields of their life. Many people do not want to admit that they have deficits in the area of reading, writing and arithmetic. The result is a retreat from social life. By renewing and refreshing the knowledge people get self-confidence and find better their place in society. This refreshing and renewing of lost knowledge takes place through hands-on learning in a group, without coercion, but with a lot of fun. The recovered confidence and fun in a group allows many people to participate in society again.

To 2: We can increase the incentives for adults because we use games that are fun and can be played without much knowledge of every human being. The combination of games and the subsequent learning motivation is increased to deal with a subject matter, which one would not normally do. So that is the motivating feature of gambling is exploited in order to learn a difficult subject matter.

To 3: We can offer tailored learning opportunities to individual learners through games, in such a way that we appropriately choose the games to the participants. It can be dealt with cultural differences as well as differences in learning behaviour and knowledge. E.g. if we have a

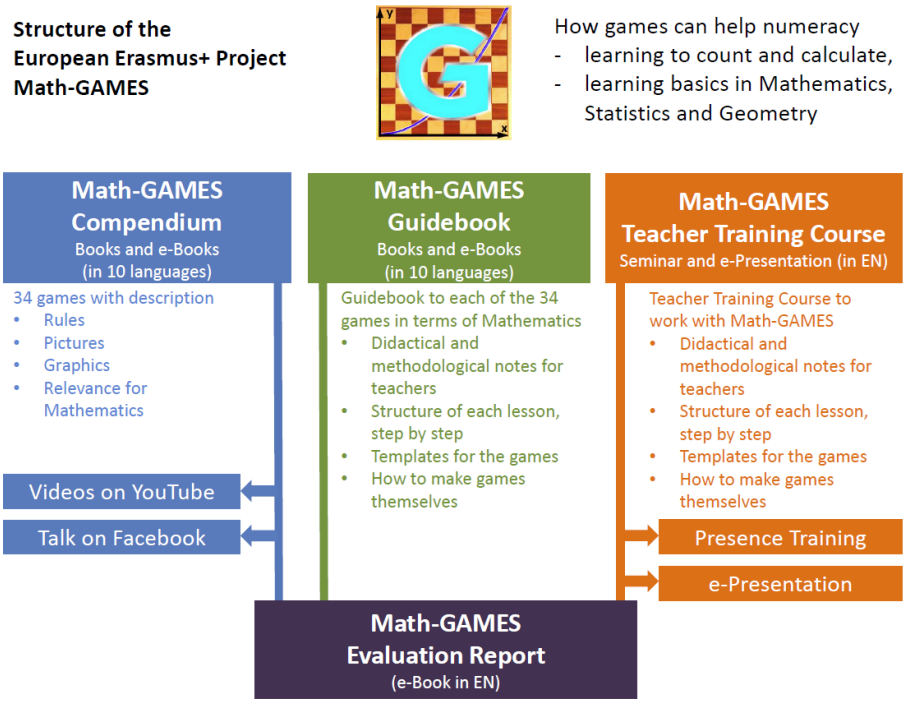


group of adult migrants from Arabia, people can be reached by Tafli game faster than with a card game which is unknown. We can also choose to watch so that mathematical knowledge is targeted, such as accurate identification of numbers between 1 and 6 at dice. But for instance, if a group wants to learn everything for commercial arithmetic, we can support learning with fun and enjoying the game "Monopoly".

To 4: We can provide information on accessing to the services of adult education by creating a low input threshold so that everyone loses his inhibitions to attend courses for adults. By announcing that it is a class with games participants will come who would otherwise never visit a mathematical course.

To 5: In different countries, we can protect traditional and popular games from being lost, because we use only those games that are known and are used by many people. This helps to save the game from extinction, because many people only play with electronic devices and no longer traditional games. Moreover, these games are more suitable for learning, as the social component is larger and provides more fun.

Structure of the European Erasmus+ Project Math-GAMES



The four parts of the output of the project Math-GAMES:

1. **Math-GAMES Compendium** of Famous Traditional Games, which are books in ten languages (BG, DE, EN, ES, FR, GR, IT, RO, VA, TR). After that the partners of the project will prove, how traditional games could be implemented in their learning program for a better understanding of mathematics, especially for lower skilled people, for young people and for immigrants, if there are needs.

The results are the **Math-GAMES Numeracy Learning Guidebooks** in nine languages.

In the third part of the project the project partners will prove by doing and testing during real courses and seminars that playing games between people with different skills assist in social integration and thus traditional games will be saved by transferring them to other people and they will not be lost. The result is a **Math-GAMES Teacher Training Course and Seminar**, which is held for the next years in different countries. The e-presentation, the seminar and the teacher-training course are published in English.

Finally the **Math-GAMES Testing and Evaluation Report** is published. It is a report about the project, the work, the activities during the lessons, the competitions in schools, the meetings and the evaluation. The Math-GAMES Project Report is published in English. All the Material is available from 2018 from the website www.math-games.eu

The authors of this compendium hope now that the users will have much joy in playing our games, because joy helps you to learn. In addition the authors hope to make a contribution that more people can apply basic mathematical content through this compendium.

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THE MATH-GAMES METHODOLOGY

MATHEMATICS AND THE ROLE OF GAMES IN LEARNING AND TEACHING - WHY USING GAMES IN LEARNING MATHEMATICS?

by Andreas Skotinos, Cyprus

The Goals of Mathematics and its Centrality in an Adult's Life

It is generally agreed that doing mathematics is a critical skill for all, adults and children, geniuses and people with limited intelligence, persons with high education and individuals with low literacy and knowledge.

In quite a number of reports it is recognized and stressed that in order for adults to function (reasonably well) in an increasingly complex world, they require a basic level of numeracy, which is increasingly necessary in a range of life-skills, such as personal finance and data handling. It is also accepted that mathematical skills (at least at the very elementary level) are increasingly needed in the workplace and in everyday transactions between people.

It is not by chance that Aeschylus, 25 centuries ago, in the "Prometheus Bound" is adding that besides the fire, which Prometheus gave to people, he points out *"And yes, I invented for them numbers, too, the most important science"**. This reveals the close relation of humans to mathematical literacy and their need to develop mathematical skills, at least at the elementary level.[1]

These basic skills are obviously reflected in the main goals of mathematics education, which are to prepare students to:

- Solve problems
- Communicate and reason
- Make connections between mathematics and its applications
- Become mathematically literate
- Appreciate and value mathematics
- Make informed decisions as contributors to society.

As can be seen the majority of these goals are immediately related to general life skills, that are expected for any adult and consequently it is justifiable to promote the learning of this subject to any person irrespective of his/ her ability and degree of intelligence.

The Role of Games in Learning Mathematics

So we have to promote Mathematics learning by any means. In view of this need the question now becomes "How can Games promote the Learning of Mathematics?" Particularly this question becomes more important in the case of Slow Adults Learners. The Background that can support a successful promotion of Games in the learning process can stem out of expectations that can have positive impact on the following aspects of human behavior: **cognitive, motivational, emotional and social**. Existing research, although not extensive as yet, supports this positive impact. Particularly in the case of Slow Adults Learners the positive impact on the motivational, emotional and social aspects is crucial and it is expected to have positive influence on the cognitive aspects as well.

In Psychology it is recognized that Play brings joy. And it is vital for problem solving, creativity and relationships. This is true for every person either a child or an adult. In particular this is vital for slow learners as it is one of the very few sources to provide these elements, while for other adults there may be other sources as well. Furthermore research in Psychology relates Play with much social behavior that we want either to enhance (if they are directing to the right direction) or to diminish (if they are leading to the wrong direction).

For instance, a psychologist found that lack of Play was just as important as other factors in predicting criminal behavior among murderers in Texas prisons.

Thus when we are dealing with the use of Games in the processes of teaching (and hence learning) we would better employ techniques and methods aiming at:

- **Creating Interest and Promoting Motivation**
  A Game is a sequence of interesting choices. By engaging the learner in such a process motivation is

[1] Prometheus was chained to a high rock as punishment because he brought salvation to man, by stealing the gods' fire, but also by giving man numbers and their meaning. Thus, already 2,500 years ago, Aeschylus in his "Prometheus Bound" confirms the **importance of numbers for mankind**.

https://www.youtube.com/watch?v=kcWdcGwd844

activated and thinking (including critical one) is taking place.

- **Utilizing the Benefits That Games Provide in Engaging Learners in an Environment of Experiential and Active Learning**
  The interaction in a game creates a better understanding for the learners in regards to the objects, concepts, processes and even the other learners involved
- **Socializing the Persons Involved and Exploiting the Competition and Challenge Element**
  Games are part of everyday life-socialization. This is particularly important in the case of slow learners as their slowness might have its roots in their lack of social relations and interchanging eliminating ideas or low morale.
- **Connecting to Real Life Situations**
  Quite many games reflect actual activities of life and thus they provide the element of usefulness.
- **Developing a Happy and Joyful Environment**
  As already mentioned the joy element is a plus in the learning process.
- **Utilizing the Design (Structure, Rules, Equipment, Problem Posing etc.) of a Game in Order to Develop an Appropriate Learning Approach**
  The components of a game, particularly the ones characterized by aesthetic, illustrative, energetic activities can be exploited for meaningful learning. Also the problem solving elements provide ample ideas for strategic and critical thinking.

The Math-Games Methodology

The Math-Games methodology encompasses a series of activities that will provide a teacher (and in particular a teacher of slow learners adults) the background for using Games as an educational medium in developing mathematical literacy. In this context it includes three main outcomes (a Math-Games Compendium, a Math-Games Guidebook and a Math-Games Teacher Training Course) that support various approaches and methods for learning and teaching.

Factors that are to be considered in adopting the Math-Games methodology

In designing a lesson through the adoption of the Math-Games methodology and considering that the main target group of learners is going to be slow adult learners it is useful to take into consideration a number of factors reflecting some of the possible difficulties of these learners. The effort will be to exploit the power of Games in order to alleviate or diminish these difficulties. Such factors include the following ones:

- **Language Issues**
  In mathematics classes, language problems are evident when students have trouble using symbols of math, expressing math concepts to others, and listening to mathematics explanations. Problems also appear in expressing math “sentences”.
- **Cognitive Factors**
  These may be attributed to perceptual, memory, attention or reasoning factors. Perception involves taking in information from the environment and processing that information for storage or use.
- **Metacognitive Factors**
  Metacognition is an awareness of the skills, strategies, and resources that are needed to perform a task and the ability to use self-regulatory mechanisms, including adjustments, to complete the task. Students with metacognition problems have trouble selecting and using effective learning strategies. Games could provide the forum to face such difficulties.
- **Motor Factors**
  Motor skills, like perceptual ones, involve more than one process. They may involve memory of the symbol along with its actual formation (visual and motor memories). They may involve visual perception and transfer (copying). Or they may involve integration of fine muscles with task demands. Indicators of motor problems are highly visible: poorly formed symbols, little control of spacing, excessive time for a task, and avoidance of written work.
- **Social and Emotional Factors**
  Such factors cover a very broad spectrum including peer relations, cooperation, self-esteem etc. Games again could provide a medium for facing them.
- **Habits of Learning**
  “Habits of learning” refers to how individuals view and participate in learning, their self-discipline and self-motivation, goal setting, engagement in learning activities, and acceptance of challenges.
- **Previous Experiences**
  Particularly in the case the learner had negative previous experience he refuses to get involved in the learning process. Games again could alleviate such negative experiences.

Consequently what we have to take into consideration in designing our plan for teaching using the Math-Games methodology could be summarized as:

- Type of slow learner (is the slowness due to other learning areas?)
- Background in Math
- Socialization needs of the person
- Motivation needs and indications that the mathematical content of the game relates to everyday life
- Provision of opportunities for the use of the previously mentioned benefits

General Approaches for Using Games in Learning Mathematics

Obviously the approach one will adopt for using Games in the learning process depends on a number of goals that we want to achieve ranging from the mathematical area or topic to the considerations mentioned just in the

previous paragraphs, reflecting the benefits of the methodology. In this context we can suggest the following approaches:

- **Using the Methodology as an Introduction to a Mathematical Topic**
  The idea is to ask the learners to play a game that can be associated with the learning objectives of the particular game. Playing a game can be used as a brainstorming. This idea is expected to be the basis for motivation and developing of interest. It can also be used as an icebreaker both for the relations of the people involved in the learning process (learners and teacher) and for the attitudes of the learners towards mathematics (which are usually negative).
- **Using the Methodology for Creating a Happy and Joyful Environment**
  This idea develops positive conditions for learning and thus overcoming negative attitudes and anxiety.
- **Using the Methodology as an Actual Educational Medium for Comprehension of Mathematical Concepts and Processes**
  Obviously such an approach is a substitute for a more traditional one with the advantage that it exploits the benefits of the methodology.
- **Using the Methodology for Consolidation of Otherwise Learned Concepts or Processes**
  It is a fact that learning process, particularly for mathematics, demands such an approach.
- **Using the Methodology for Relating Mathematics to Real Life Situations**
  The identification of uses of mathematics for real life situation is an asset for adults as the need to see applications of what they have to learn.
- **Using the Methodology for Developing Problem Solving and Critical Thinking Skills**
  It is a major goal that every learner develops such skills. Games are ideal for strategic thinking, planning and designing approaches to face problematic issues. It provides the forum for meaningful learning and not just rote memorization.
- **Using the Methodology for Boosting Creativity, Productivity and Innovation**
  This idea enhances the skills of the learners and provides a fruitful approach for learning. It can be utilized for adaptation of games or constructing new ones by the players.
- **Using the Methodology for Fixing Relationship Difficulties among the Learners**
  As mentioned earlier such an approach can create a cooperative, challenging and joyful environment, thus creating ideal conditions for learning.

The following table indicates some examples for the various approaches that are presented in this Guidebook:

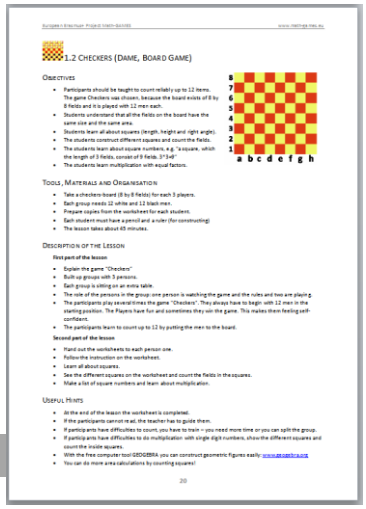
Approach	Game presented in the Math-Games Compendium and in the Guidebook
Introduction to a Topic	1.2 Checkers
Joyful Environment	4.1 Petanque
Educational medium	1.3. Damath, 10.1 Okey, 3.2 Math Scrabble
Consolidation	10.3 Sudoku
Math in Real Life	3.3 Monopoly
Problem Solving and Critical Thinking	2.3 Combination 9, 7.1 Magic Square, 9.3 Nim-Game
Creativity, Productivity, Innovation	1.4 Tangram, 8.2 Skipping Rope
Fixing Relationships	5.2 Seven Steps, 8.3 Hora

HINTS TO USE THIS GUIDEBOOK

- **The objective of the guidebook is to provide teachers and educators with material to help them teach basic skills in mathematics.**
- **The guidebook includes 33 games.**
- **The best way for the teachers to choose the one that better suits them is to check the Synopsis (p. 10) where there is a list of the games and also the mathematical content that is associated with each one.**
- **Each section of the guidebook is dedicated to one game.**

Structure of this Book - How to use this Guidebook?

Each chapter of this Guidebook consists usually of

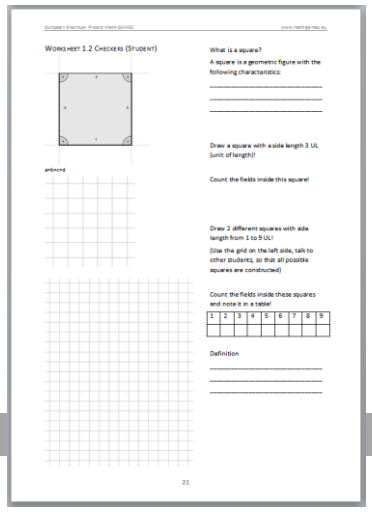


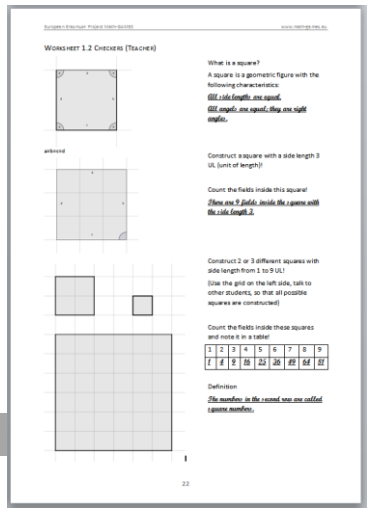
Preliminary Remarks on the Lesson

with objectives, hints for tools, materials and organisation, with description of the lesson and with other useful remarks, to read by the teacher when preparing the lesson

Worksheet for the Learners

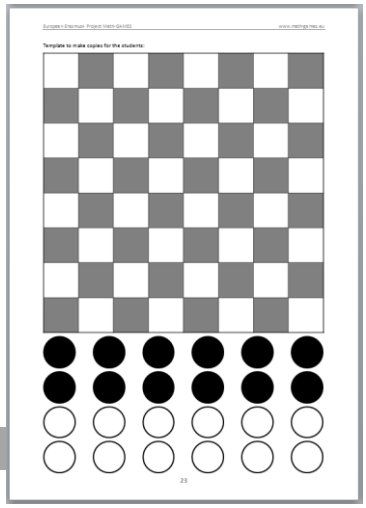
with gaps and free areas to be completed by the student during the lesson





Worksheet for the Teachers

as a basis for the teacher during the lesson



Copying templates and other material

with which the teacher can prepare his lesson more easily

SYNOPSIS

Page 1 (Game 1 to 17): In this Synopsis of all possible learning objectives for the beginner's lesson in Mathematics (in the first column), the learning objectives are marked with an **X** which can be methodically covered with the mentioned games.

Name and number of the game	1.1 Nine Men's Morris - Mill	1.2 Checkers	1.3. Damath	1.4 Tangram	1.5 Sea Battle	1.6 Connect Four	1.7 Dice Games	1.8 Domino	2.1 Skambalove	2.2 Do not get angry - Ludo	2.3 Combination 9	3.1 Black Jack	3.2 Math Scrabble	3.3 Monopoly	4.1 Petanque	4.2 Tictactoe	4.3 Rock-Paper-Scissors
Area of Mathematics (A) with Objectives	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
A1 Whole numbers																	
Count reliably up to 10 items	X	X	X	X	X	X	X	X	X	X	X				X	X	X
Draw a numberline and sort numbers up to 10	X					X										X	
Understand a coordinate system and find positions up to 10					X	X				X	X				X	X	
Count reliably up to 20 items			X		X	X		X	X	X	X				X	X	X
Read and write numbers up to 10, including zero			X						X	X	X				X	X	
Read and write numbers up to 20, including zero															X	X	
Draw a numberline and sort numbers up to 20																X	
Understand a coordinate system and find positions up to 20		X			X												
Order and compare numbers up to 10, including zero	X								X	X	X				X	X	
Order and compare numbers up to 100, including zero									X	X	X	X	X	X	X	X	
Understand a coordinate system and find positions up to 100								X									
Add single-digit numbers with totals to 10	X		X				X					X	X	X			
Add two-digit whole numbers									X		X		X				
Subtract single-digit numbers from numbers up to 10			X					X	X	X							
Recall addition and subtraction facts to 10															X		
Interpret +, - and = in practical situations for solving problems			X									X	X	X	X		
Multiply using single-digit whole numbers		X	X									X	X	X			
Use a calculator to check calculations using whole numbers	X	X	X				X	X				X	X	X			
Approximate by rounding to the nearest 10																	
Use and interpret +, -, x, and / in practical situations			X												X		
Order and compare numbers up to 20, including zero								X	X		X		X		X		
Recognize, describe, and extend patterns								X							X	X	
Determine a next term in linear patterns (e.g., 2, 4, 6...)									X								
A2 Fractions, decimals and percentages																	
Read, write and compare halves and quarters of quantities				X													
Find halves and quarters of small numbers of items or shapes				X													
Read and add halves of quantities.																	
A3 Common Measures																	
Recognize and select coins and notes												X	X	X			
Make amounts of money												X	X	X			
Relate familiar events to the year, month and week																	
Describe size and use comparisons															X		
Describe length, width and hight		x			X										X	X	
Describe weight and use it									X								
Read and understand time displayed digital									X								
Understand weight measure									X						X		
Read and understand temperature																	
A4 Shape and Space (Geometry)																	
Recognize and name two dimensional shapes		X		X	X	X		X							X	X	X
Describe length and width of shapes		X		X	X										X	X	X
understand line, line section, distance						X									X	X	
Recognize and name three dimensional shapes							X								X	X	
Describe length, width and hight of shapes							X		X	X	X				X	X	
Understand common everyday names of shapes									X						X		
Understand symmetry in shapes																	
Understand side length, length of a line section		X		X		X											
Understand and compare angles															X		X
Measure areas by counting squares or using grids		X		X	X						X						
Work out simple volumes							X		X								
A5 Data and statistical measures																	
Understand simple informations from lists, tables and diagrams											X						
Sort and classify objects using a single criterion											X						
Construct simple diagrams																	
A6 Probability																	
Understand probability					X					X	X						X
Identify the range of possible outcome when using a dice										X							
Identify the range of possible outcome when using more dices																	
Identify the range of possible outcome when using a card																	
Identify the possibilities of success each time we remove a match																	X
A7 Mathematical reasoning																	
Determine the approach, materials and strategies to be used				X								X	X	X	X	X	X
Use tools, such as manipulative or patterns, to solve problems				X								X		X	X		
Observe rules (game rules or mathematical rules)	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

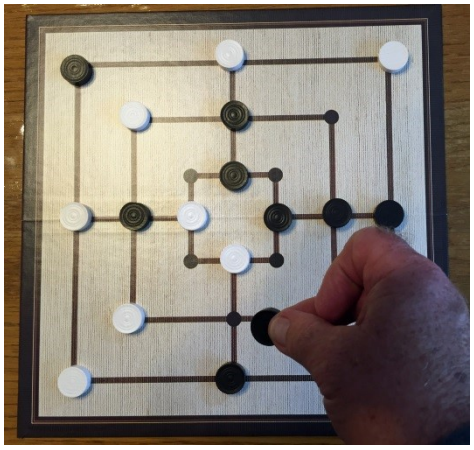
SYNOPSIS

Page 2 (Game 18 to 34): In this Synopsis of all possible learning objectives for the beginner's lesson in Mathematics (in the first column), the learning objectives are marked with an **X** which can be methodically covered with the mentioned games.

Name and number of the game	5.1 Ludo	5.2 seven steps	6.1 Backgammon	6.2 Chess	6.3 Calculator Hopscotch	7.1 Magic Sqare	7.2 Four Seasons	7.3 Steel the pile	8.1 Crows feet - Cat's Cradle	8.2 Skipping Rope	8.3 Hora Romanian Dance	9.1 Fifeteen-Game Puzzle	9.2 Seven and Half	9.3 Nim Game	10.1 Okey - Rummikub	10.2 Hopscotch	10.3 Sudoku
Area of Mathematics (A) with Objectives	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
A1 Whole numbers																	
Count reliably up to 10 items	X	X	X	X	X	X	X	X	X	X	X			X		X	X
Draw a numberline and sort numbers up to 10		X	X	X			X		X	X	X					X	
Understand a coordinate system and find positions up to 10	X	X	X	X					X	X	X					X	
Count reliably up to 20 items		X		X	X	X	X	X	X	X	X	X	X				
Read and write numbers up to 10, including zero		X		X	X		X		X	X	X					X	X
Read and write numbers up to 20, including zero		X			X	X	X		X	X	X	X	X				
Draw a numberline and sort numbers up to 20		X							X	X	X						
Understand a coordinate system and find positions up to 20		X		X					X	X	X						
Order and compare numbers up to 10, including zero		X		X					X	X	X						
Order and compare numbers up to 100, including zero						X	X	X							X		X
Understand a coordinate system and find positions up to 100				X		X											
Add single-digit numbers with totals to 10			X	X	X			X					X	X	X		X
Add two-digit whole numbers					X	X	X										
Subtract single-digit numbers from numbers up to 10					X	X			X	X	X			X			
Recall addition and subtraction facts to 10								X									
Interpret +, - and = in practical situations for solving problems					X	X	X								X		
Multiply using single-digit whole numbers			X				X								X		
Use a calculator to check calculations using whole numbers						X	X						X				
Approximate by rounding to the nearest 10																	
Use and interpret +, -, x, and / in practical situations																	
Order and compare numbers up to 20, including zero				X				X	X	X	X	X					
Recognize, describe, and extend patterns			X						X	X	X	X					
Determine a next term in linear patterns (e.g., 2, 4, 6...)			X				X		X	X	X	X					
A2 Fractions, decimals and percentages																	
Read, write and compare halves and quarters of quantities															X		
Find halves and quarters of small numbers of items or shapes																	
Read and add halves of quantities.													X				
A3 Common Measures																	
Recognize and select coins and notes																	
Make amounts of money																	
Relate familiar events to the year, month and week							X				X						
Describe size and use comparisons		X		X					X	X	X						
Describe length, width and hight		X							X	X	X						
Describe weight and use it																	
Read and understand time displayed digital																	
Understand weight measure																	
Read and understand temperature																	
A4 Shape and Space (Geometry)																	
Recognize and name two dimensional shapes	X	X		X		X			X	X	X	X				X	
Describe length and width of shapes	X											X				X	
understand line, line section, distance		X							X	X	X					X	
Recognize and name three dimensional shapes									X	X	X						
Describe length, width and hight of shapes									X	X	X						
Understand common everyday names of shapes						X	X		X	X	X					X	
Understand symmetry in shapes					X	X	X		X	X	X					X	
Understand side length, length of a line section									X	X	X						
Understand and compare angles							X		X	X	X						
Measure areas by counting squares or using grids		X							X	X	X	X					
Work out simple volumes																	
A5 Data and statistical measures																	
Understand simple informations from lists, tables and diagrams		X							X	X	X						
Sort and classify objects using a single criterion								X	X	X							
Construct simple diagrams									X	X	X						
A6 Probability																	
Understand probability			X				X	X	X	X			X	X			
Identify the range of possible outcome when using a dice			X														
Identify the range of possible outcome when using more dices			X				X										
Identify the range of possible outcome when using a card								X					X				
Identify the possibilities of success each time we remove a match									X	X				X			
A7 Mathematical reasoning																	
Determine the approach, materials and strategies to be used						X	X	X	X	X	X	X			X		X
Use tools, such as manipulative or patterns, to solve problems												X			X		X
Observe rules (game rules or mathematical rules)	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

1.1 Nine men's morris - Mill (Board Game)

Objectives



- Participants should be taught to **count reliably up to 9 items**. The game Mill was chosen, because number 9 is the biggest single digit number.
- They understand that if items are rearranged the number stays the same.
- They know how to count on and back from any small number.
- They learn the idea of a **number line**.

Tools, Materials and Organisation

- Take millboard for each 3 players.
- Each group needs 9 white and 9 black men.
- Prepare copies from the worksheet for each student.
- The lesson takes 45 minutes.

Description of the Lesson

First part of the lesson

- Explain the game "Nine men's Morris"
- Arrange groups with 3 persons.
- Each group is sitting on an extra table.
- The role of the persons in the group: one person is watching the game and two persons are playing.
- The participants play several times the game "Nine Men's Morris". They always have to begin with 9 men. The Players have fun and they win the game sometimes. This makes them feeling self-confident.
- The participants learn to count up to 9 by putting the men to the board.

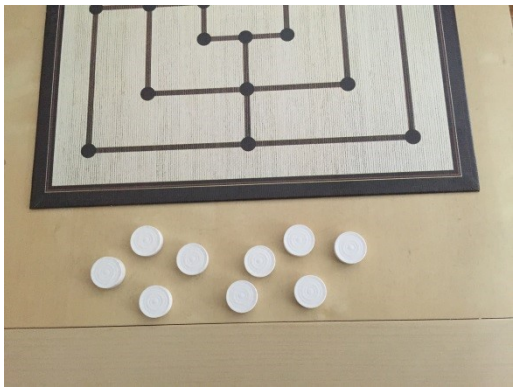
Second part of the lesson

- Hand out the worksheets to each person one.
- Follow the instruction on the worksheet.
- Mark the men with numbers.
- Students learn that the last number when counting is the number, which represents the amount of the men.
- The participants learn that numbers have an order and built up a number line.
- Add "0" to the order and the number line.

Useful Hints

- At the end of the lesson the worksheet is completed.
- If the participants cannot read, the teacher has to guide them.
- If participants have difficulties to count, you have to train – you need more time or you can split the group.
- If participants have difficulties to write numbers, you have to divide the lesson into two lessons: First lesson: play and count, second lesson: play and write numbers.
- Following lesson: Look for another game, in which the participants have to count up to 9.

WORKSHEET 1.1 (LEARNER)



You start the game Nine Men's Morris with 9 men for each player.

How many men do you see on the picture?

_____

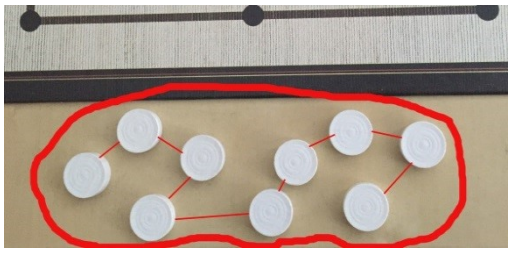
Count the men again and write numbers on it!

What is the last number?

_____

_____

_____



Give each man a number according to the lines! Start on the left side!

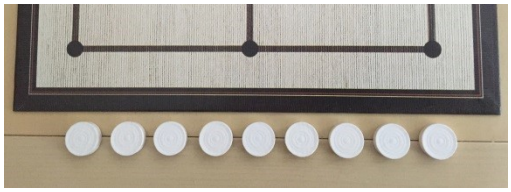
The last number is the amount of the men.

How many men are all together?

_____

_____

_____



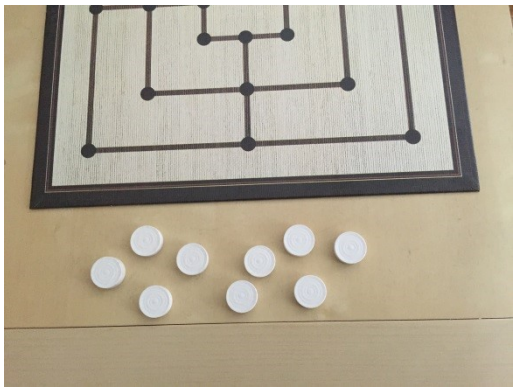
Give each point on the line a number!

You have got a _____

and the numbers are _____,

they are in an order!

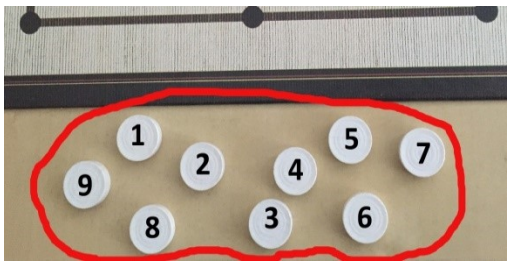
WORKSHEET 1.1 (TEACHER)



You start the game Nine Men's Morris with 9 men for each player.

How many men do you see on the picture?

I see 9 men!

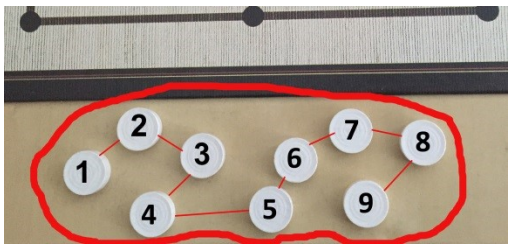


Count the men again and write numbers on it!

What is the last number?

1 2 3 4 5 6 7 8 9

The last number is 9!



Give each man a number according to the lines! Start on the left side!

The last number is the amount of the men.

How many men are all together?

9

because the last number during counting is 9

Give each point on the line a number!

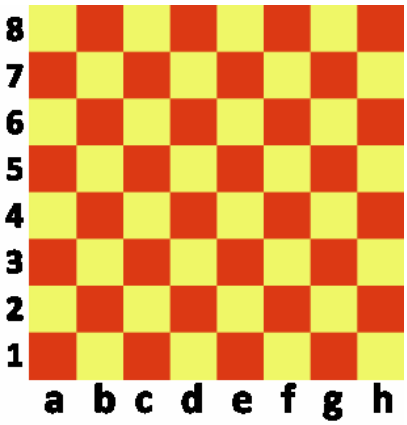
You have got a ***number line***

and the numbers are ***sorted***,

they are in an order!

1.2 Checkers - Dame (Board Game)

Objectives



- Participants should be taught to count reliably up to 12 items. The game Checkers was chosen, because the board exists of 8 by 8 fields and it is played with 12 men each.
- Students understand that all the fields on the board have the same size and the same area.
- Students learn all about squares (length, height and right angle).
- The students construct different squares and count the fields.
- The students learn about square numbers, e.g. “a square, which the length of 3 fields, consist of 9 fields. 3*3=9”
- The students learn multiplication with equal factors.

Tools, Materials and Organisation

- Take a checkers-board (8 by 8 fields) for each 3 players.
- Each group needs 12 white and 12 black men.
- Prepare copies from the worksheet for each student.
- Each student must have a pencil and a ruler (for constructing)
- The lesson takes about 45 minutes.

Description of the Lesson

First part of the lesson

- Explain the game “Checkers”
- Arrange groups with 3 persons.
- Each group is sitting on an extra table.
- The role of the persons in the group: one person is watching the game and the rules and two are playing.
- The participants play several times the game "Checkers". They always have to begin with 12 men in the starting position. The Players have fun and sometimes they win the game. This makes them feeling self-confident.
- The participants learn to count up to 12 by putting the men to the board.

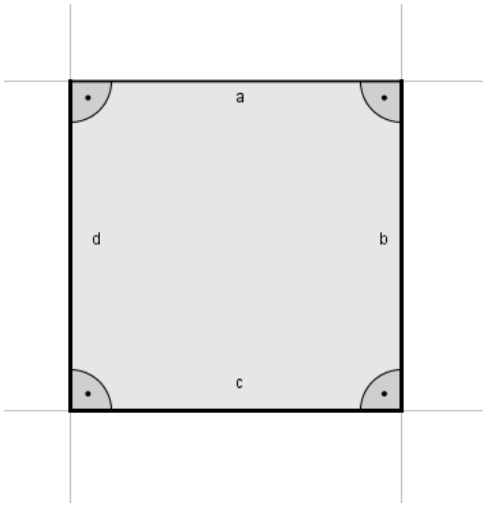
Second part of the lesson

- Hand out the worksheets to each person one.
- Follow the instruction on the worksheet.
- Learn all about squares.
- See the different squares on the worksheet and count the fields in the squares.
- Make a list of square numbers and learn about multiplication.

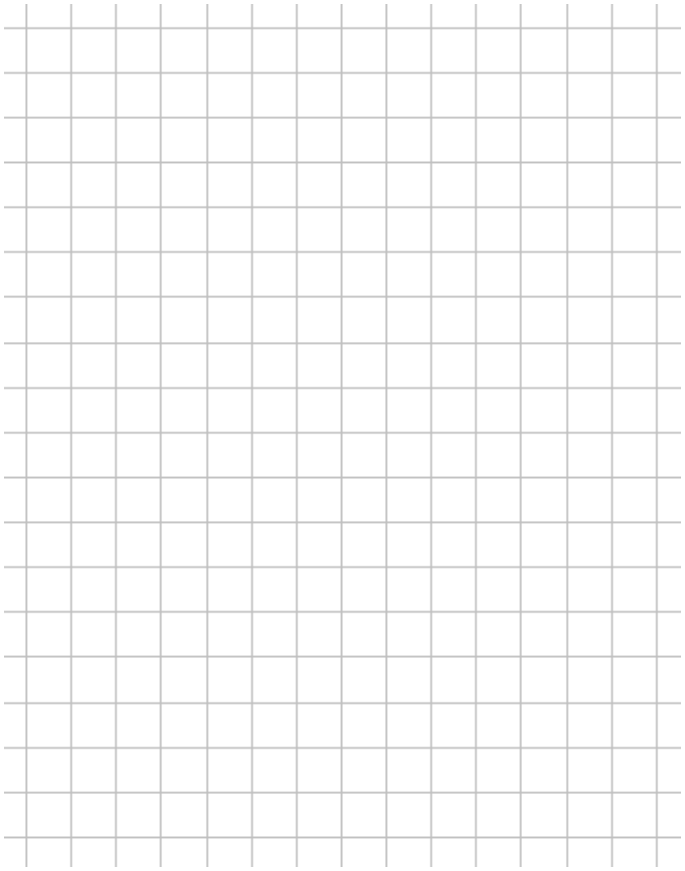
Useful Hints

- At the end of the lesson the worksheet is completed.
- If the participants cannot read, the teacher has to guide them.
- If participants have difficulties to count, you have to train – you need more time or you can split the group.
- If participants have difficulties to do multiplication with single digit numbers, show the different squares and count the inside squares.
- With the free computer tool GEOGEBRA you can construct geometric figures easily: www.geogebra.org
- You can do more area calculations by counting squares!

WORKSHEET 1.2 (LEARNER)



a=b=c=d



What is a square?

A square is a geometric figure with the following characteristics:

_____

_____

_____

Draw a square with a side length 3 UL (unit of length)!

Count the fields inside this square!

Draw 2 different squares with side length from 1 to 9 UL!

(Use the grid on the left side, talk to other students, so that all possible squares are constructed)

Count the fields inside these squares and note it in a table!

1	2	3	4	5	6	7	8	9

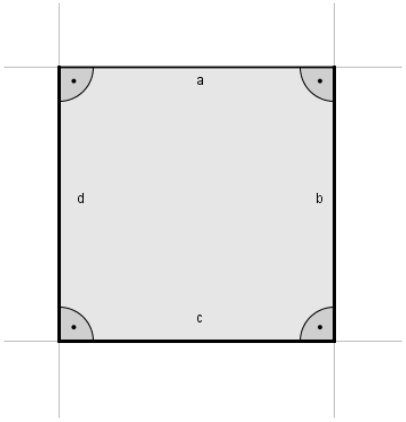
Definition

_____

_____

_____

WORKSHEET 1.2 (TEACHER)



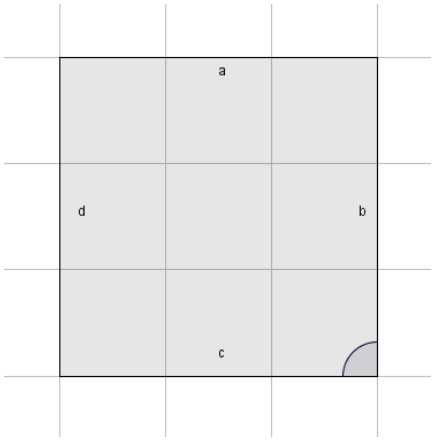
a=b=c=d

What is a square?

A square is a geometric figure with the following characteristics:

All side lengths are equal.

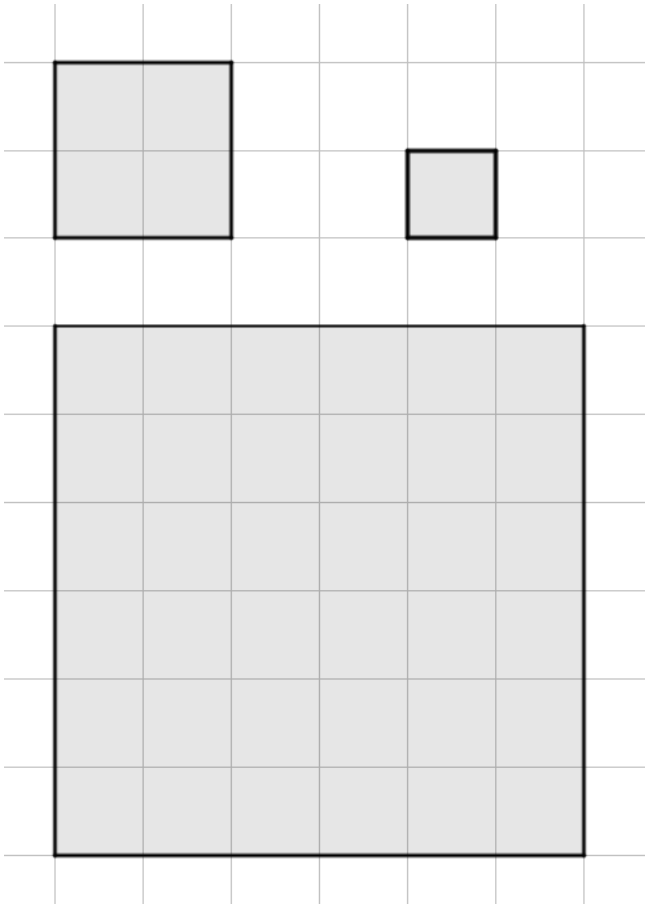
All angles are equal; they are right angles.



Construct a square with a side length 3 UL (unit of length)!

Count the fields inside this square!

There are 9 fields inside the square with the side length 3.



Construct 2 or 3 different squares with side length from 1 to 9 UL!

(Use the grid on the left side, talk to other students, so that all possible squares are constructed)

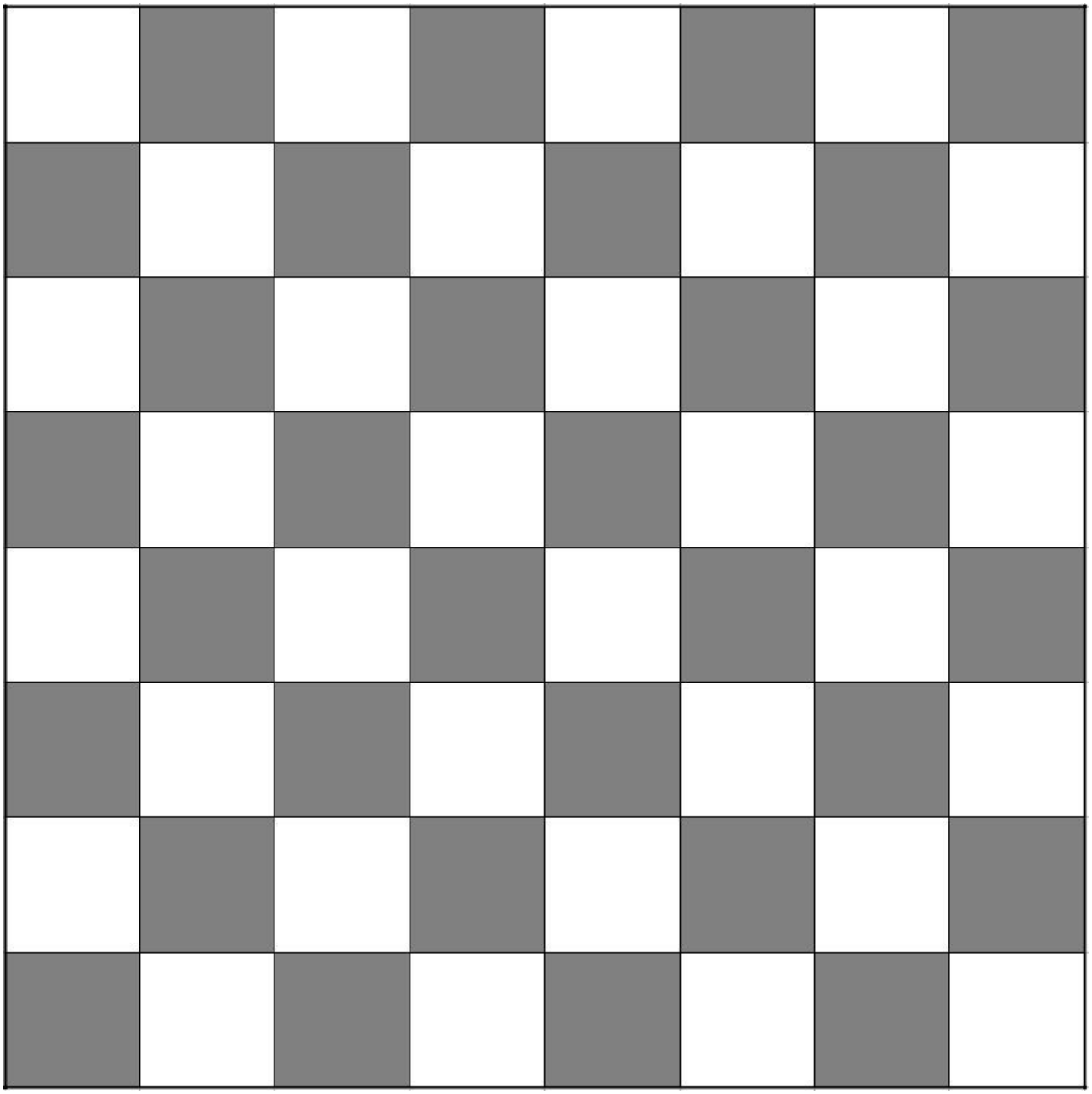
Count the fields inside these squares and note it in a table!

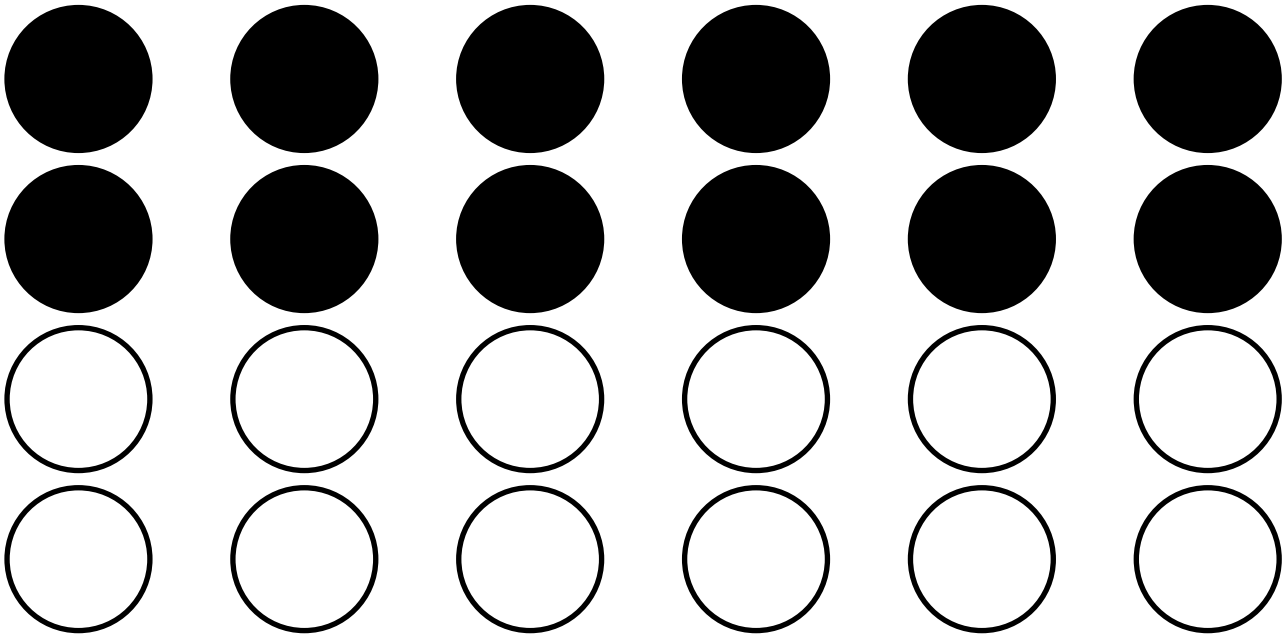
1	2	3	4	5	6	7	8	9
1	***4***	***9***	***16***	***25***	***36***	***49***	***64***	***81***

Definition

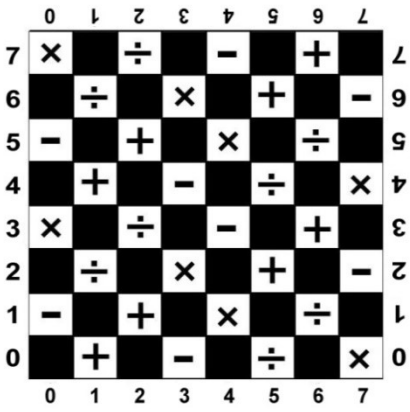
The numbers in the second row are called square numbers.

Template to make copies for the students:





1.3 DAMATH - MATH CHECKERS (BOARD GAME)

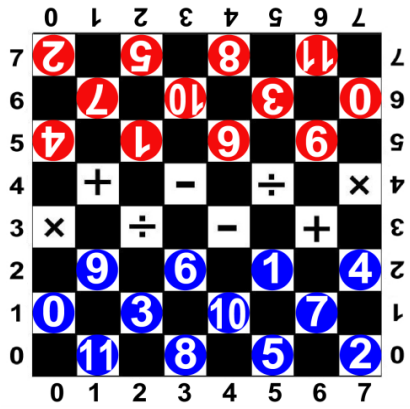


REQUIREMENTS

- Participants already can count and calculate with whole numbers between 0 and 11. Participants know, that multiplication by 0 is always 0 and divided by 0 is not possible, so this move is not allowed.
- Students already know the rules of the game Checkers (Dame)

OBJECTIVES

- The game DAMATH is a game to exercise calculating with numbers between 0 and 11.
- Students practice playing the reckoning. You have fun.



TOOLS, MATERIALS AND ORGANISATION

- Take a checkers-board (8 by 8 fields) for each 3 players.
- The white fields of the board are marked with arithmetic symbols. You can mix different symbols or, much easier, you start first just with addition and so on.
- Each group needs 12 white and 12 black men. These men are marked with whole numbers from 0 to 11.
- Use the copy template from 1.2 Checkers!
- The lesson takes 45 minutes. If the participants don't know the rules of the game Checkers, you have to play Checkers first for another 45 minutes.

DESCRIPTION OF THE LESSON

First part of the lesson (40 minutes)

- Explain the game "Damath" (the participants already know the rules of the game Checkers).
- Arrange groups with 2 persons.
- Each group is sitting on an extra table.
- The role of the persons in the group: two are playing against each other (it is a competition).
- The participants play a certain number of times the game "Damath". They always have to begin with 12 men in the starting position (see picture). The Players get only points, if they calculate in the right way when jumping over an opponent. The result of this calculation is to be noted.

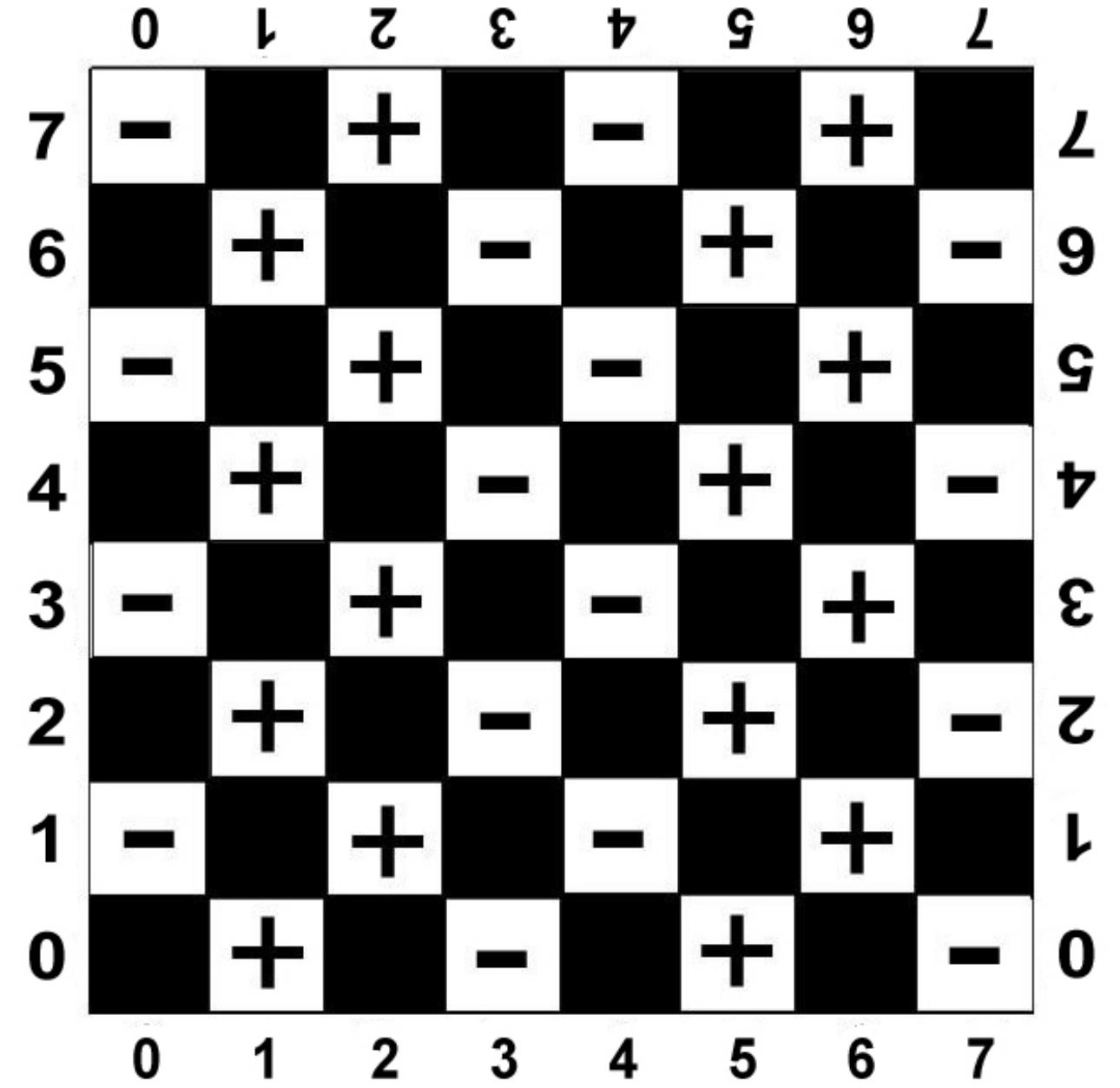
Second part of the lesson (5 minutes)

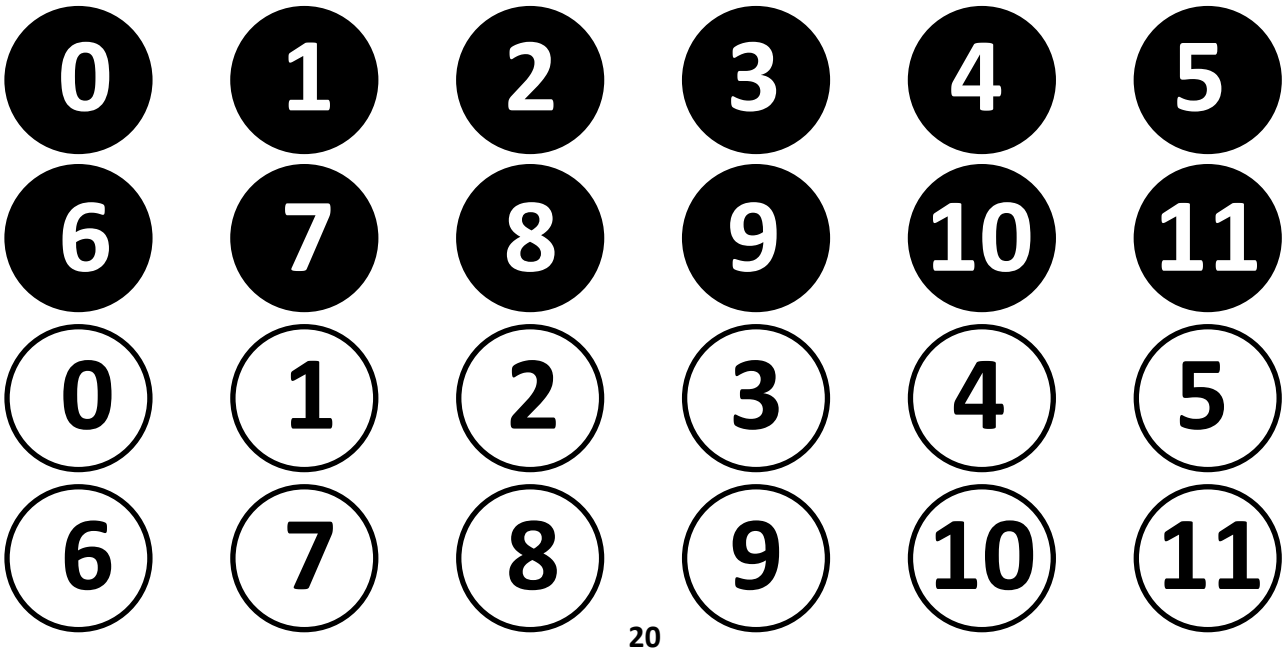
- Each participant is presenting his score.
- The winner of the group has the most scores, but everybody has learned a lot by exercising the caculations.

USEFUL HINTS

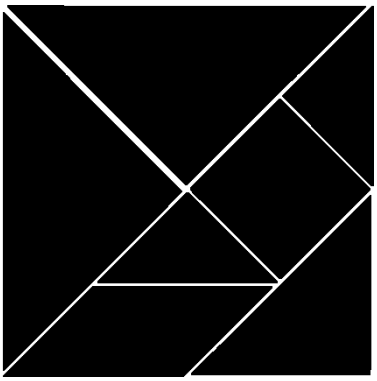
- You can construct Damath by your own (see next page).
- It is very important, that the teacher knows the abilities of the participants.
- Start, if necessary, with easier calculations, e.g. you can mark the white fields just with + and the men with numbers between 0 and 5 – so your participants calculate e.g. just 2+3=5 or 0+5=5 or 1+2=3.

Template to make copies for the students (you find more templates here: www.math-games.eu):





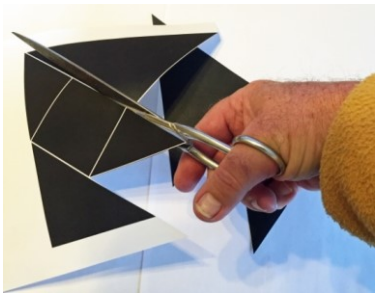
1.4 TANGRAM (PUZZLE)



OBJECTIVES

- The participants learn to construct a Tangram.
- The students learn the geometric figures triangle, square and parallelogram.
- Students learn to recognize and build geometric shapes.

TOOLS, MATERIALS AND ORGANISATION



- There are two possibilities:
  you can construct the Tangram on a strong paper by yourself or
  you copy the construction and the pupils have to use scissors to cut the figures.
- The lesson takes 45 minutes.

DESCRIPTION OF THE LESSON

First part of the lesson (20 minutes)

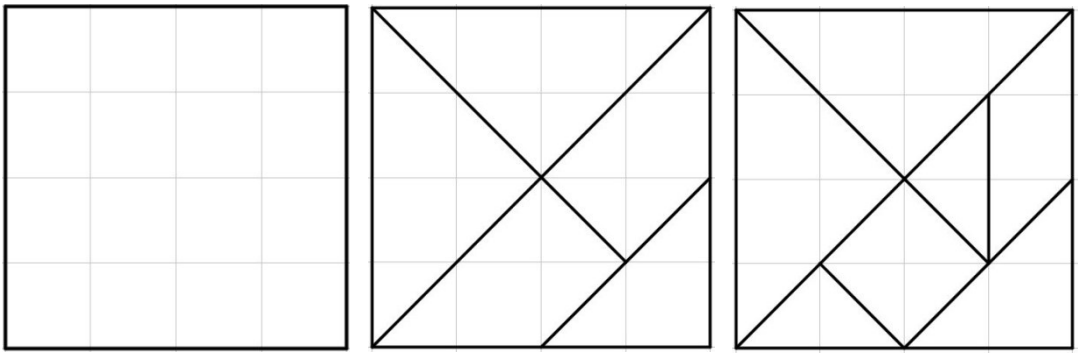
- Make groups of two participants. Each group has pencil, ruler, paper and scissors.
- Explain the Tangram and its history
- Construct the Tangram (see below, how to construct)

Second part of the lesson (25 minutes)

- Give a copy of the templates with lines to the participants (easy way).
- Give a copy of the templates without lines to the participants (not so easy).
- Each one has to puzzle about 9 templates.
- The teacher controls it and gives points to the well-done constructions.

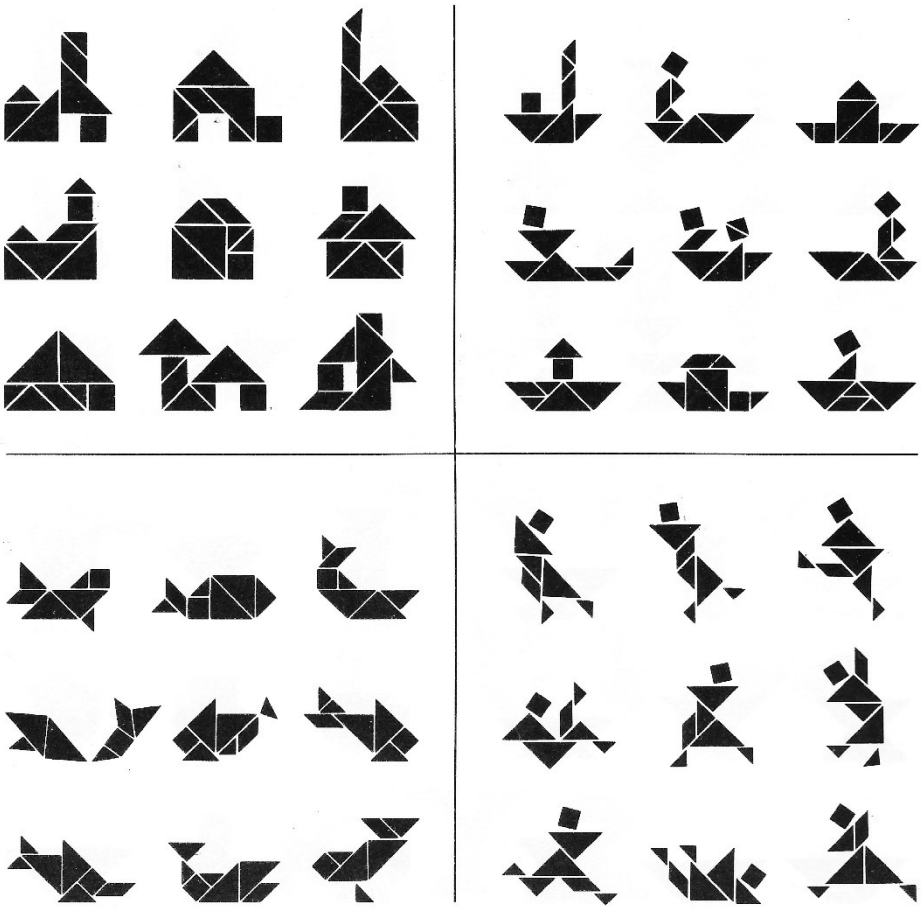
USEFUL HINTS

How to construct a Tangram by yourself:
1. Draw 1 square existing of 16 small squares
2. Draw the diagonals
3. Draw the different 7 figures of Tangram (2 small, 1 medium and 2 big triangle, 1 small square, 1 parallelogram)

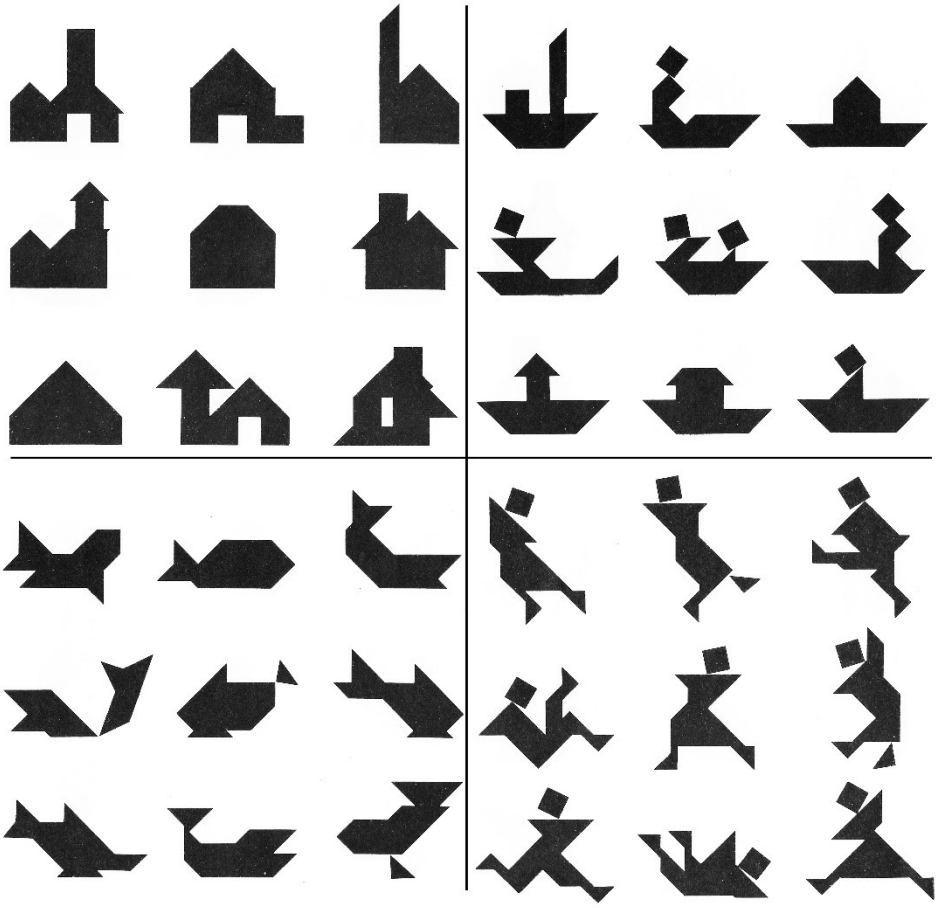


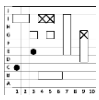
Special Feature: As a secondary task you can calculate areas by counting the squares and triangles. You will find out, that the big square has the area 16 and the 7 shapes have the areas 4, 2 and 1: 16=4+4+2+2+2+1+1

Template for puzzle-figures with lines (easy):

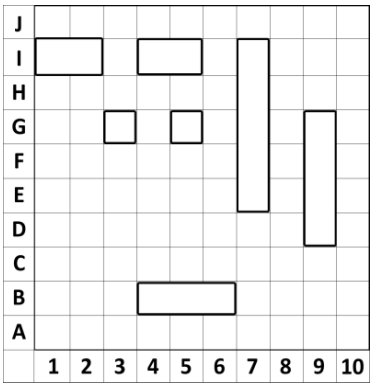


Template for puzzle-figures without lines (not easy):





1.5 Sea Battle (Paper-Pencil Game)

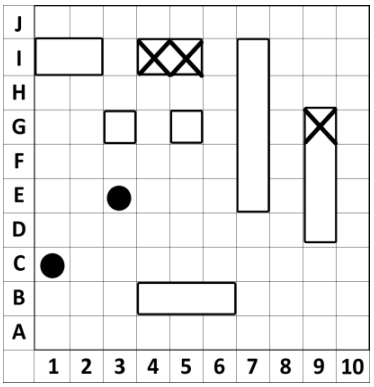


Objectives

- The participants learn to construct different rectangles (the width is always 1; the length is 1, 2, 3, 4 and 5.
- The students learn to use **game coordinates**, like 3B.
- The students learn to use **mathematical coordinates**, like (3|2).

Tools, Materials and Organisation

- Each of the two players needs two empty grids: one for to mark his own ships and one for to mark the hits own and on the enemy ships.
- The lesson takes 45 minutes.



Description of the Lesson

First part of the lesson (preparation for 5 minutes)

- Make groups of two participants. Each player has pencil and several pairs of grids.
- Each player is marking his own ships on one of the grids (1st picture).

Second part of the lesson (game 25 minutes)

- One player begins shooting by saying the coordinates.
- The other player is marking the shots on his grid and is saying: water or hit
- If one ship is full of hits, the player has to say: ship has sunk
- Go on as long as all ships of one player have sunk.

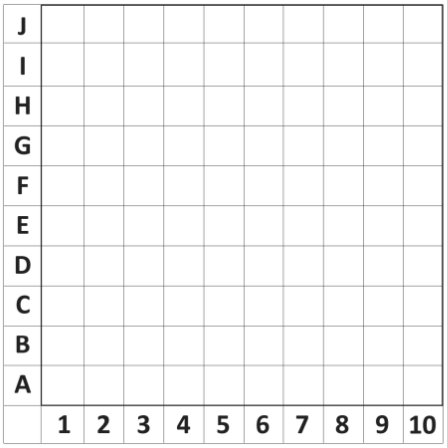
Third part of the lesson (15 minutes)

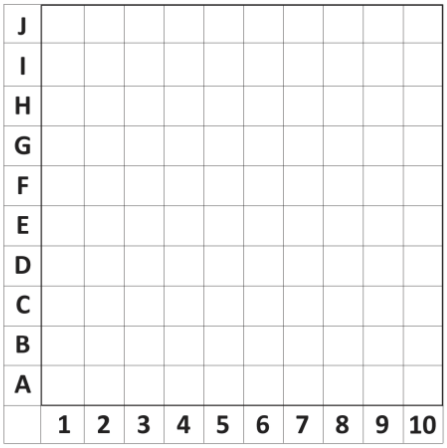
- Hand out the worksheets to each person one.
- Follow the instruction on the worksheet.
- If there is enough time, you can go back to the game.

Useful Hints

- You can do area calculations of rectangles by counting squares and/or calculating area=length*width.
- There are computer versions available, where you can play “Sea Battle” alone against the computer:
  http://de.battleship-game.org/
  http://en.battleship-game.org/
  http://es.battleship-game.org/

Template





WORKSHEET 1.5 (LEARNER)

What is a grid?

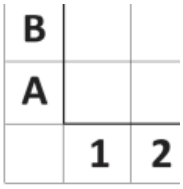
A grid is a geometric background with the following characteristics:

There are _____ and _____ lines. All lines have the _____ distance from each other.

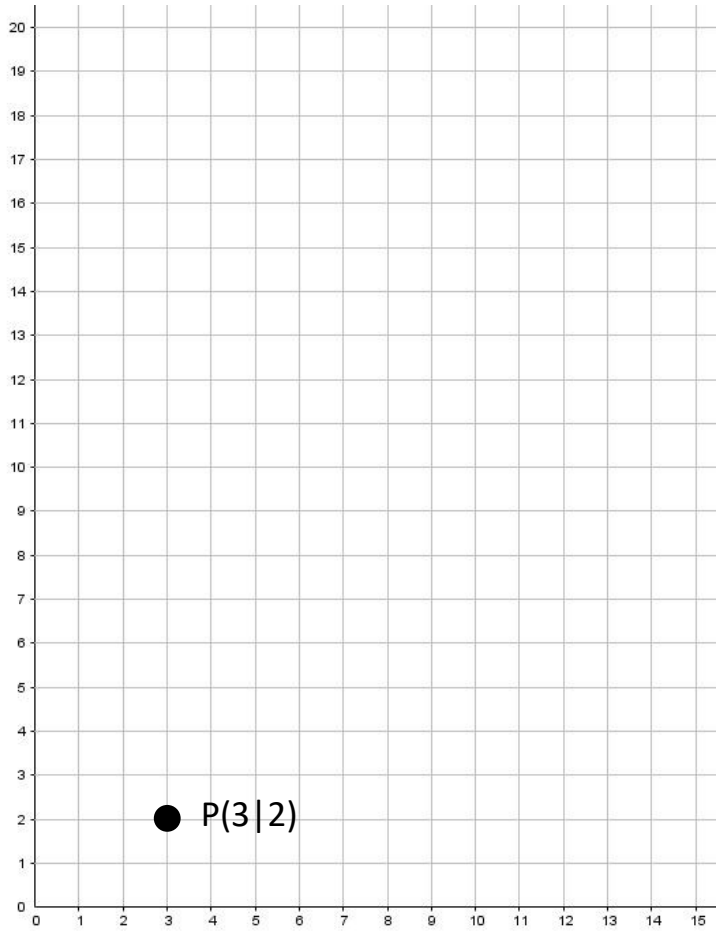
Complete the grid on the left side!

Give each column a number from 1 to 10 and each row a letter from A to J (this is a 10 x 10 grid).

Try to locate the field 2D (column 2, row D) = (2|D) and mark more fields: (1|A), (10|C), (10|J), (5|J)



Definition: (2|D) are the _____ of the field in column 2 and row D.



In Mathematics not the fields, but the lines and its crossing-points are marked. Coordinate System on the left side with P(3|2)

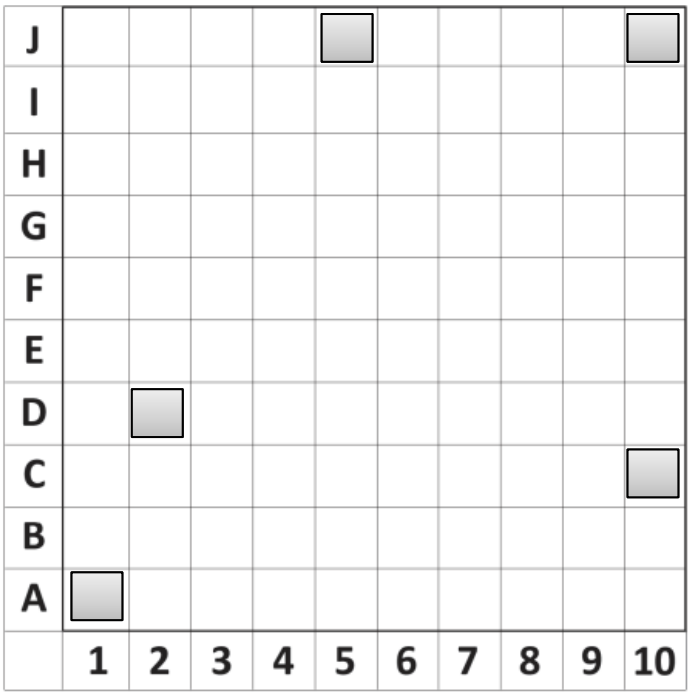
Definition: (__|__) are the math. co-ordinates of the crossing point of column-line __ and row-line __.

Mathematical notation: _____

Put more points into the math. co-ordinate system and add the mathematical notation!

A(1|1), B(__|__), C(__|__), D(__|__)

WORKSHEET 1.5 (TEACHER)



What is a grid?

A grid is a geometric background with the following characteristics:

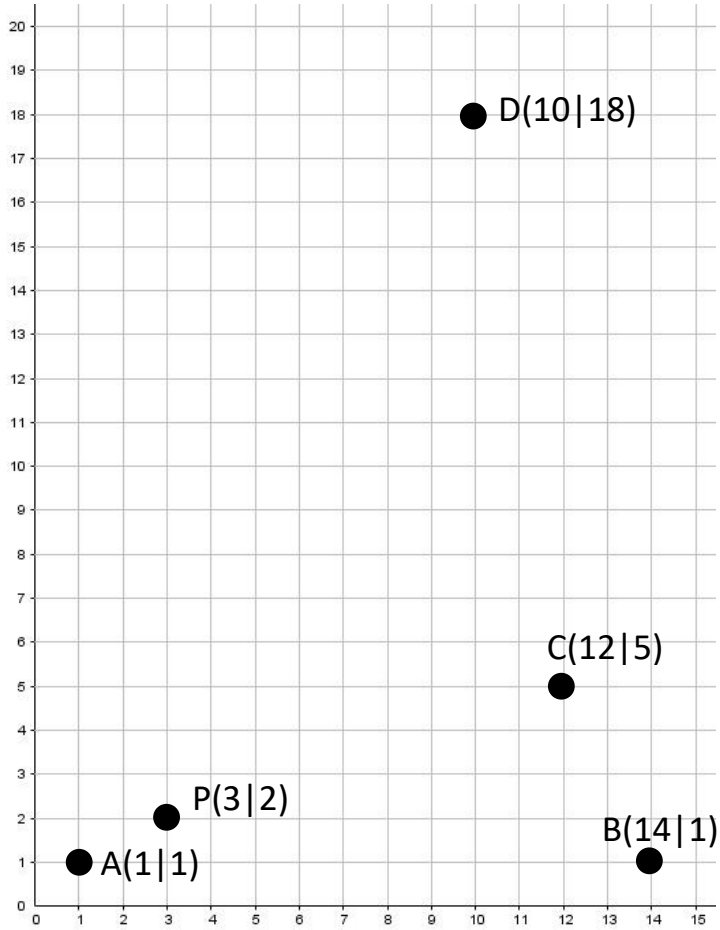
There are horizontal and vertical lines. All lines have the same distance from each other.

Complete the grid on the left side!

Give each column a number from 1 to 10 and each row a letter from A to J (this is a 10 x 10 grid).

Try to locate the field 2D (column 2, row D) = (2|D) and mark more fields: (1|A), (10|C), (10|J), (5|J)

Definition: (2|D) are the ***game coordinates*** of the field in column 2 and row D.



In Mathematics not the fields, but the lines and its crossing-points are marked. Coordinate System on the left side with P(3|2)

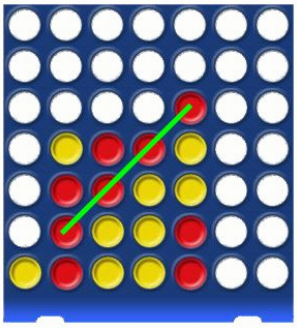
Definition: (***3|2***) are the math. ***co-ordinates*** of the crossing point of column-line ***3*** and row-line ***2***.

Mathematical notation: ***P(3|2)***

Put more points into the math. co-ordinate system and add the mathematical notation!

A(1|1), B(***14|1***), C(***12|5***), D(***10|18***)

1.6 Connect Four (Board Game)



Objectives

- The participants learn to construct a line by 2 points.
- The students learn that 4 points can be the same line or not.
- The participants determine the approach, materials and strategies to be used in this game

Tools, Materials and Organisation

- Each of the two players needs a "Connect Four" board. If you don't have enough boards, you can use instead of a board a sheet of paper as a simulation of the board (see below).
- The lesson takes 45 minutes. The students can do the second part of the lesson only, if they have learned about coordinates in lesson 1.5

Description of the Lesson

First part of the lesson (25 minutes)

- Make groups of two participants.
- Always two players have
  - either an original board with chips in two colours or
  - two pencils in two colours and a game template (see below).
- Play the game several times. Discuss about the strategies.

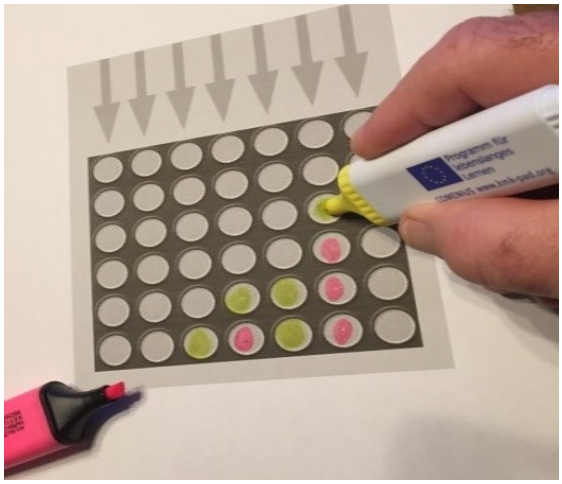
Second part of the lesson (20 minutes)

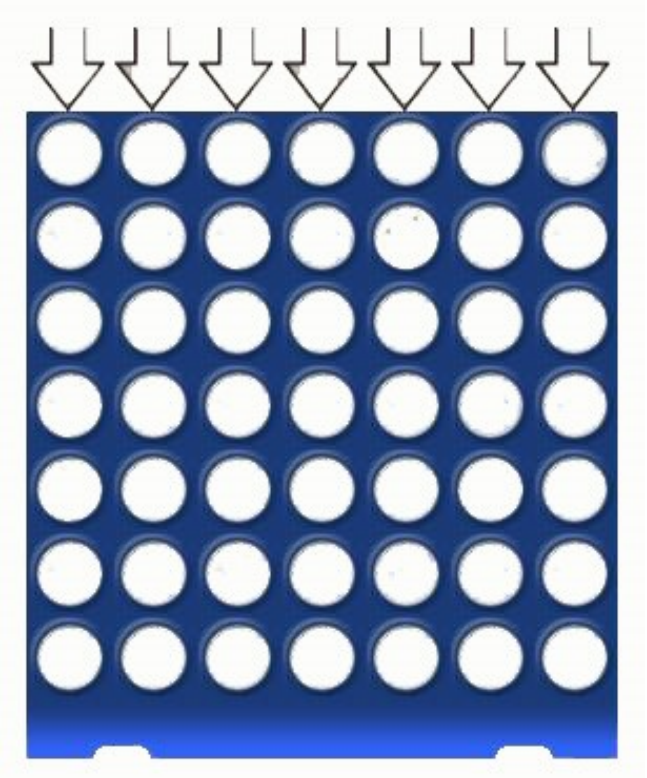
- Hand out the worksheets to each person one.
- Follow the instruction on the worksheet.
- If there is enough time, you can talk about the differences between two and three dimension games.

Useful Hints

Here you see the simulation of the game, if you don't have an original board.

Please be aware, that you fill the rows without a gap from the top in direction of the arrow!





WORKSHEET 1.6 (LEARNER)

What is a distance line?

A distance line is a geometric element with the following characteristics:

The _____ determinse a distance line.

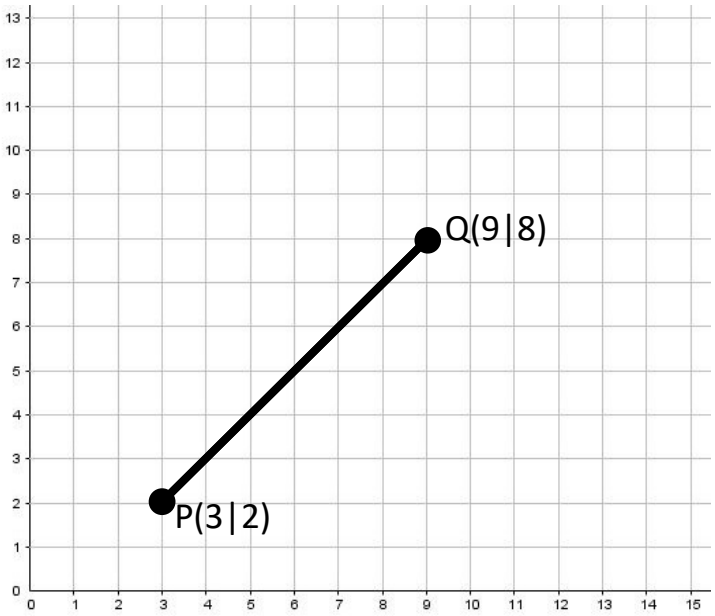
Construct in the grid on the left side two points with the coordinates P(3|2) and Q(9|8)!

Take a ruler and a pencil and connect these two points by a straight line.

The line is the _____ between two points.

_____

_____

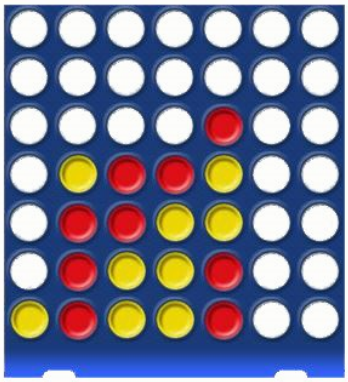


Plot more points in the painting on the left side: R(5|4), S(3|7), T(7|6) and U(10|4).

What characteristics do these points have?

_____

_____

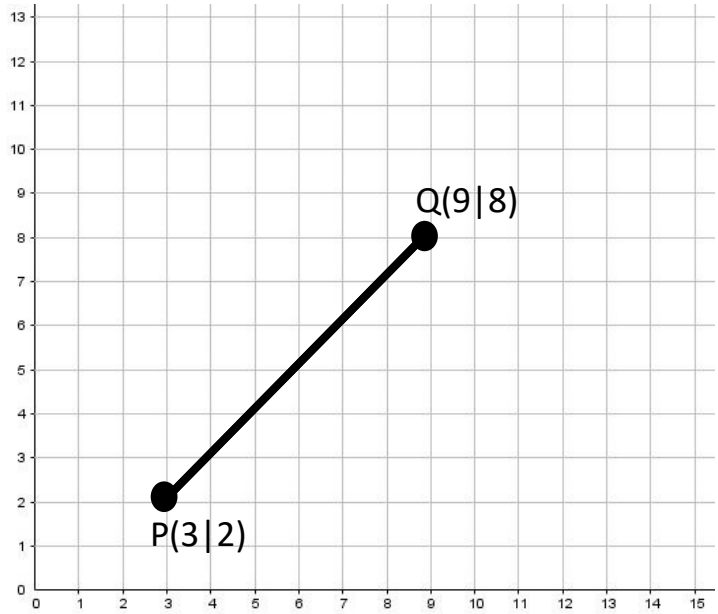


You win the game “Connect Four”, if you can put

_____

in a line!

WORKSHEET 1.6 (TEACHER)



What is a distance line?

A distance line is a geometric element with the following characteristics:

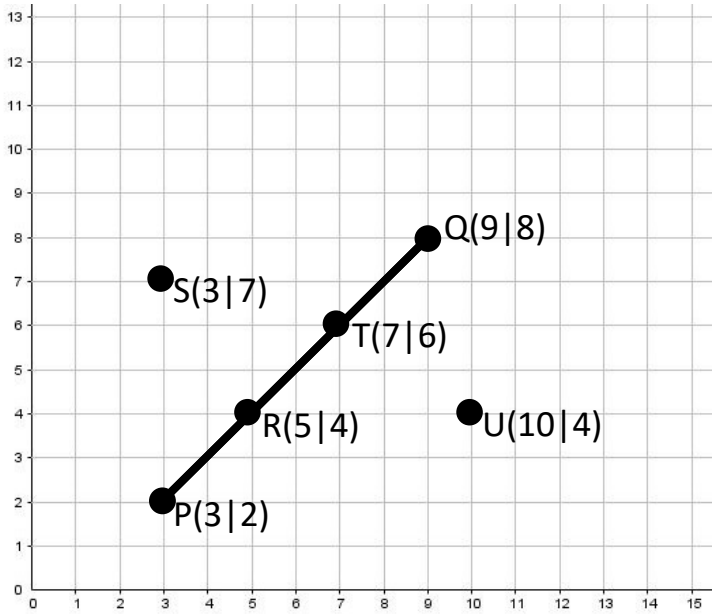
The ***shortest connection of two points*** determines a distance line.

Construct in the grid on the left side two points with the coordinates P(3|2) and Q(9|8)!

Take a ruler and a pencil and connect these two points by a straight line.

The line is the ***shortest*** connection between two points.

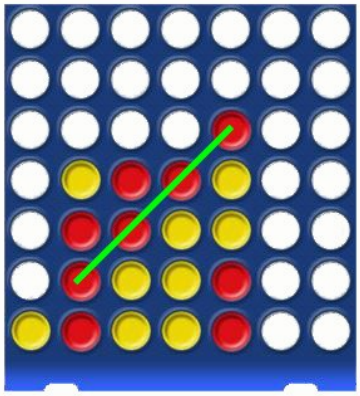
As the line is bounded by the points P and Q one says "line segment".



Plot more points in the painting on the left side: R(5|4), S(3|7), T(7|6) and U(10|4).

What characteristics do these points have?

Points R and T are on the line segment, points S and U not.



You win the game "Connect Four", if you can put

4 of your own chips = 4 points

in a line!

1.7 Ten Simple Dice Games (Table Game)

Objectives

- The participants learn all about a dice.
- The participants learn to construct a cube from a net of a cube.
- The students learn how a cube is transformed into a gambling dice.

Tools, Materials and Organisation

- Groups of 3 to 5 persons are sitting around a table.
- On each table are three dice, a shaker, notepaper and a pencil.
- For the construction of a cube you need paper, scissors and glue.

Description of the Lesson

First part of the lesson (20 minutes)

- Make groups of 3 to 5 participants.
- Choose two from the compendium of 10 dice games
- Play the game several times and note the winner.
- Which game is the most funny or exciting one?

Second part of the lesson (25 minutes)

- The participants construct their own cube or dice from a net of a cube.
- Each participant gets one worksheet, ruler and pencil, scissors and glue.
- Follow the instruction on the worksheet.

Useful Hints

It is necessary, that each participant is constructing his own cube and dice – this is learning by doing!

Interesting fact: The sum of two opposite sides of the points of a dice is always 7.

WORKSHEET 1.7 (LEARNER)

What is a cube?

A cube is a geometric object with the following characteristics:

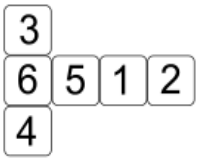
The cube is _____

The cube is bounded by _____

_____

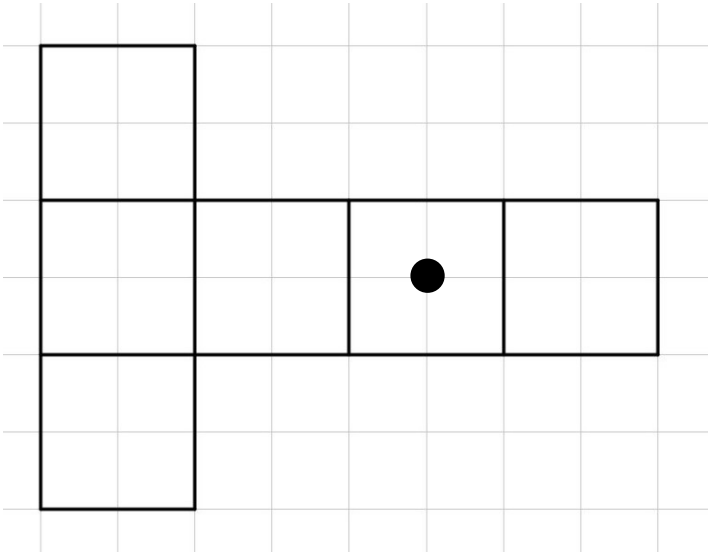
Construct in the grid on the left side the net of a cube with ruler and pencil!

Take scissors and cut out the net of the cube.



Fold along the line to make a cube!

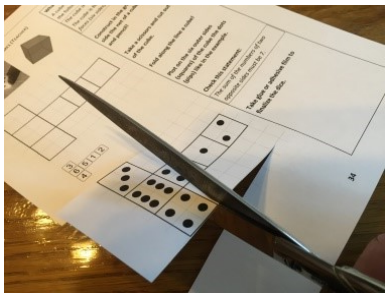
Plot on the six outer sides (squares) of the cube the dots (pips) like in the example.

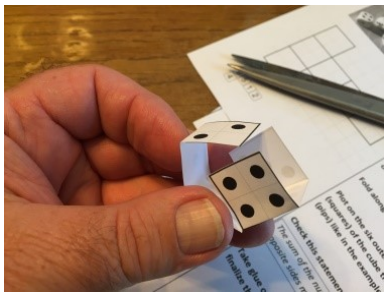


Work order: Add the points of two opposite sides!

The sum of the points of two opposite sides must be _____.

Take glue or adhesive film to finish the dice.





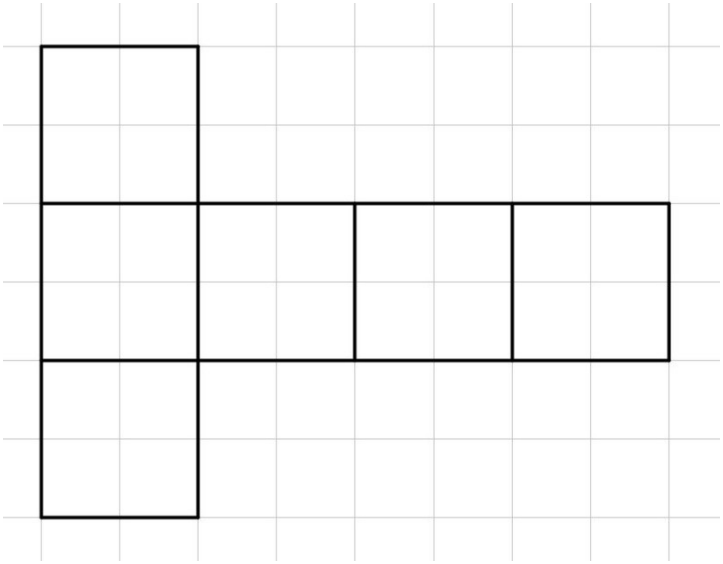
WORKSHEET 1.7 (TEACHER)

What is a cube?

A cube is a geometric object with the following characteristics:

The cube is three dimensional.

The cube is bounded by six square faces (six square sides).

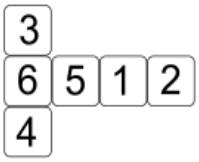


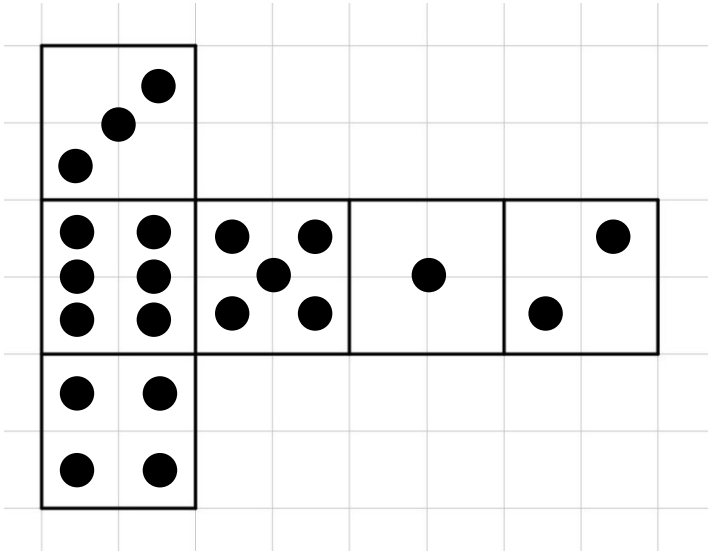
Construct in the grid on the left side the net of a cube with ruler and pencil!

Take scissors and cut out the net of the cube.

Fold along the line to make a cube!

Plot on the six outer sides (squares) of the cube the dots (pips) like in the example.

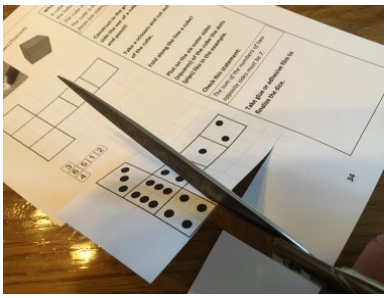


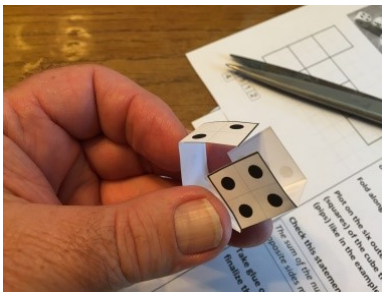


Work order: Add the points of two opposite sides!

The sum of the points of two opposite sides must be always 7.

Take glue or adhesive film to finish the dice.





1.8 DOMINO (TABLE GAME)

OBJECTIVES

- The participants learn all about dominos as rectangle game pieces.
- The participants learn to construct a domino on the paper.
- The students learn to count from 0 to 9 and to visualize these numbers.

TOOLS, MATERIALS AND ORGANISATION

- Groups of 2 to 4 persons are sitting around a table.
- On each table there is one set of dominos.
- For the construction of a set of dominos from paper you need
    - a copy of this page and scissors to cut or
    - paper, ruler, pencil to construct and scissors to cut.

DESCRIPTION OF THE LESSON

First part of the lesson (15 minutes)

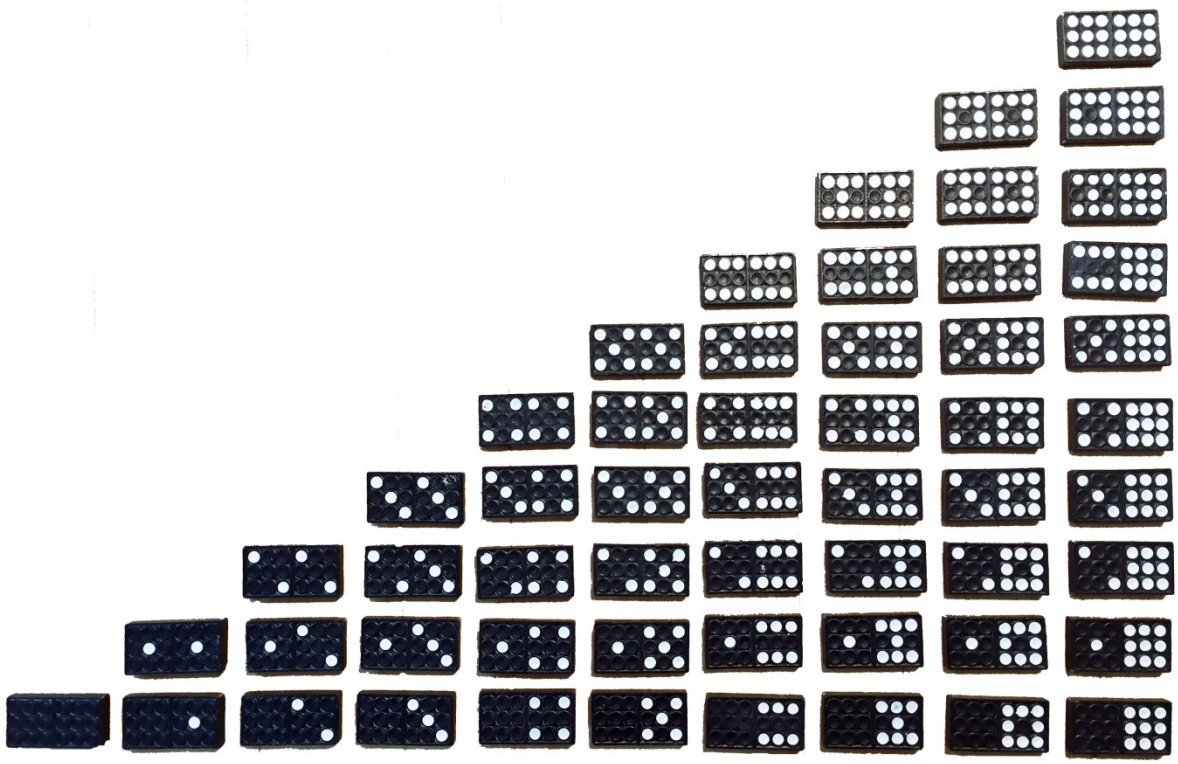
- The participants construct their own set of 55 dominos.
- During the construction they learn to count and to visualize the numbers from 0 to 9
- Each participant has paper, ruler and pencil to construct and scissors to cut.

Second part of the lesson (30 minutes)

- Make groups of 3 to 5 participants.
- Play the domino game several times and note the winner.

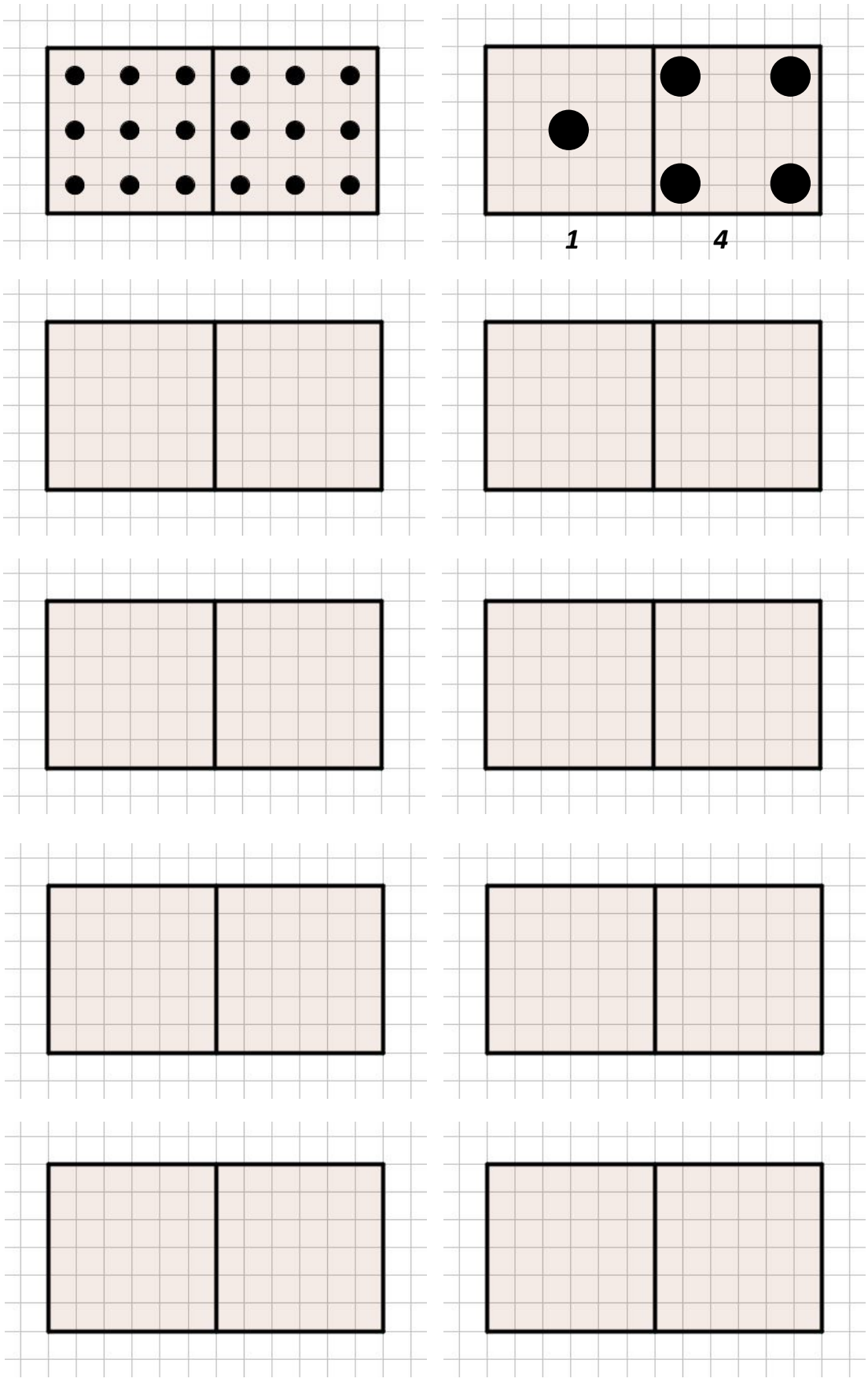
USEFUL HINTS

It is necessary, that each participant is constructing his own domino – this is learning by doing!
Especially the visualizing of numbers is important.



Template 1.8 to Make Copies for Learners

If you want to construct a full 55 domino set, you need to copy 7 of these templates.
It is important, that the participants can visualize the numbers from 0 to 9 by their own.
Please write the numbers below the stone!



2.1 SKAMBALOVE (MARBLE GAME)

OBJECTIVES

- Counting from 1 to 36
- Arrangement and comparison of numbers to 36
- Summing one-digit and two-digits number
- Introduction to geometric figures rectangle, circle and sphere

TOOLS, MATERIALS AND ORGANISATION

- Each player gets one marble
- Draw the field for the game

DESCRIPTION OF THE LESSON

First part of the lesson

- Game area is a rectangle shape on the ground with parameters 3x4m
- Each player chooses a marble
- The teacher explains the rules and the game begins
- Setting the turn of the players

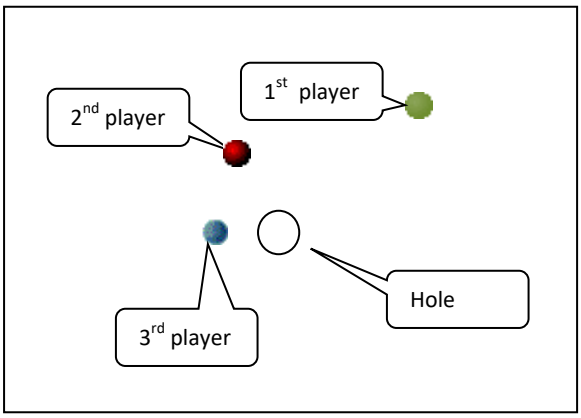
Second part of the lesson

- Hand out the worksheets, one to each person
- Follow the instruction on the worksheet

USEFUL HINTS

- If the participants cannot read, the teacher has to guide them
- If participants have difficulties to count to 36, you have to train – you need more time or you can split the group
- The participants have to train to recognize geometric shapes rectangles, circle and sphere
- The participants have to train to sum numbers in steps by 3 up to 36

WORKSHEET 2.1 PAGE 1 (LEARNER)



The players are chosen. How many are they (see the picture middle left)?

_____

How many marbles does each player have at the beginning?

_____

Show the order of play.

_____

If the first player hits the hole, how many points does he get?

_____

If the second player hits the hole two times, how many points does he get?

_____

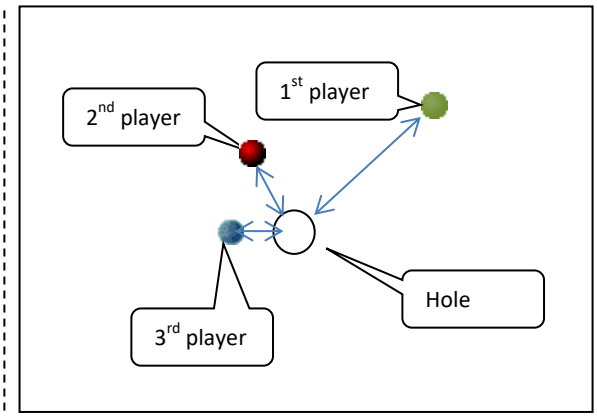
If the third player hits the hole two times and hits a marble of other player, how many points does he get?

_____

How many times a player has to hit the hole or another marble to win the game?

_____

WORKSHEET 2.1 PAGE 1 (TEACHER)



The players are chosen. How many are they (see the graphic middle left)?

3

How many marbles does each player have at the beginning?

one

Each player tosses the marble towards the hole, aiming to move it as close as possible to it or inside of it. The player who tosses the marble the closest to the holes starts the first.

Make the turn of the players!

3 2 1 because of the distance to the hole.

If the first player hits the hole, how many points does he get?

3

If the second player hits the hole two times, how many points does he get?

6

If the third player hits the hole two times and hits a marble of other player, how many points does he get?

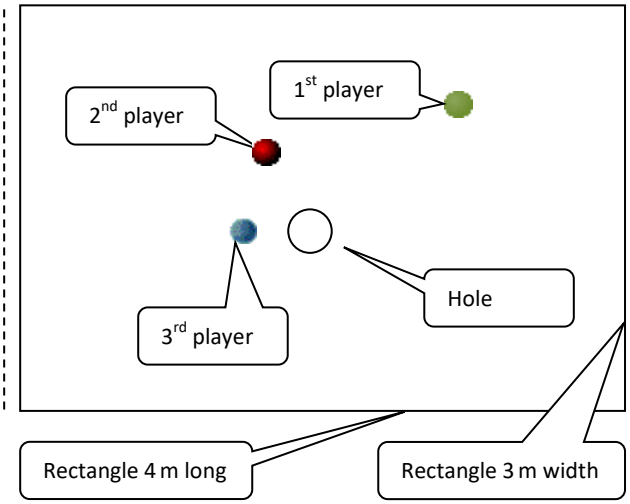
9

How many times a player has to hit the hole or another marble to win the game?

12

The winner is the player who gets 36 points first.

WORKSHEET 2.1 PAGE 2 (LEARNER)



Describe the shape of the game field?

_____

Calculate the perimeter of the game field?

_____

Calculate the area of the game field?

_____

Advanced Exercise

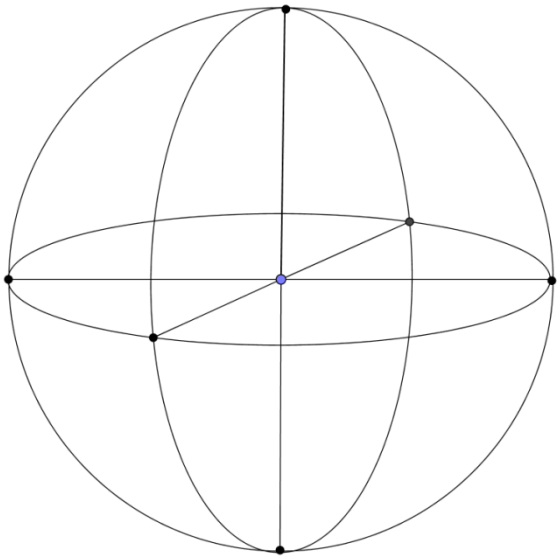
Each player gets one marble.

There are 5 red, 6 blue and 7 yellow marbles in a bag. If you have your eyes closed how many marbles must you subtract in order to be sure that you have at least 2 with a different color?

a) 4

b) 18

c) 8



Questions for the very clever ones

Describe the shape of the marble?

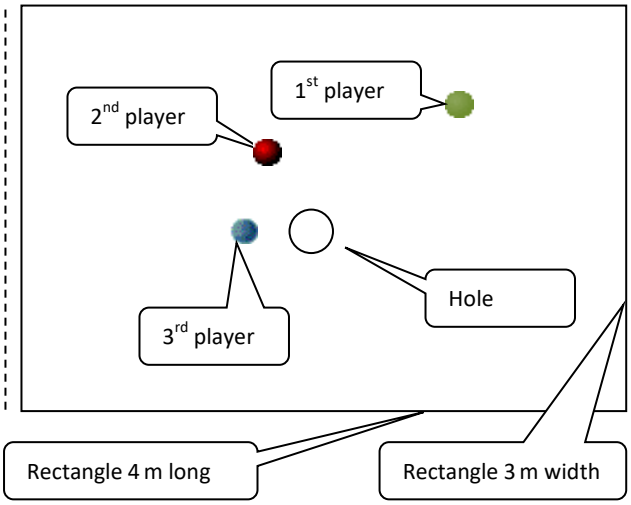
_____

Calculate the volume of the sphere, if

$\pi = 3{,}14$ and radius r=3 cm ?

_____

WORKSHEET 2.1 PAGE 2 (TEACHER)



Describe the shape of the game field?

rectangle

Calculate the perimeter of the area?

3m + 3m + 4m + 4m = 14 m

Calculate the area of the game field?

3m x 4m = 12 m²

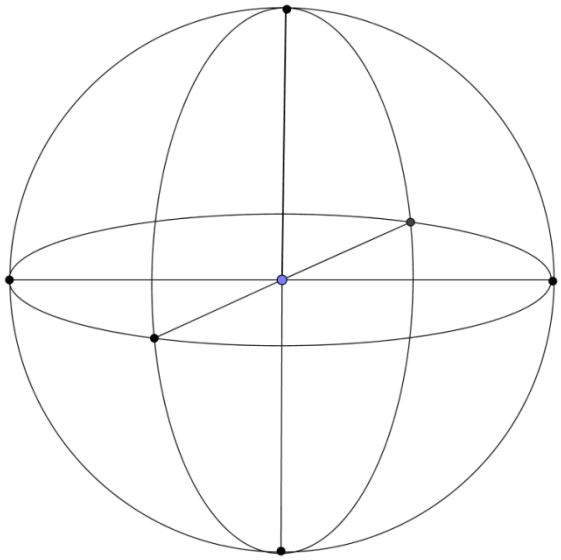
Advanced Exercise

There are 5 red, 6 blue and 7 yellow marbles in a bag. If you have your eyes closed how many marbles must you subtract in order to be sure that you have at least 2 with a different color?

a) 4 b) 18 c) 8

Solution

Seven marbles of one color are the most to be subtracted; the eighth will be mandatorily of different color.



Questions for the very clever ones

Describe the shape of the marble?

The shape of the marble is a sphere

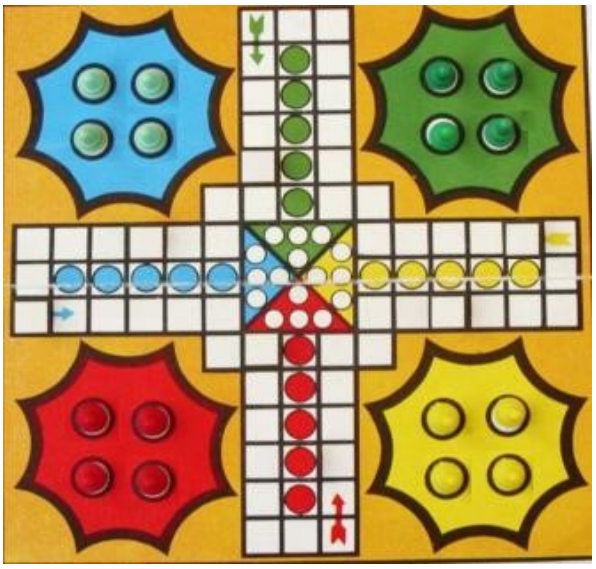
Calculate the volume of the sphere, if

$\pi = 3{,}14$ and radius r=3 cm ?

$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\, 3^3 \text{cm}^3 = 113{,}04\ \text{cm}^3$

2.2 DO NOT GET ANGRY - LUDO (BULGARIA)

OBJECTIVES:



- Introduction to geometric figures cube, rectangle, square
- Knowledge of numbers from 1 to 6 of each side of a dice
- Count reliably from 1 to 10, learn to count to 60
- The participants also learn basics in probability theory

TOOLS, MATERIALS AND ORGANISATION

- Take a game-board and dice
- Each player needs 4 pawns
- Prepare copies from the worksheet for each student
- The lesson takes 90 minutes

DESCRIPTION OF THE LESSON

First part of the lesson

- Up to four players must be chosen
- Each player chooses the color of his four pawns and places them at the dashboard start field
- Determining the sequence of the players
- The participants review the rules and the games begins
- Players are getting to know the numbers from one to six and also the six sides of the geometric figure cube
- While rolling the dice participants learn that the highest number they can get is 6 and the lowest is 1.

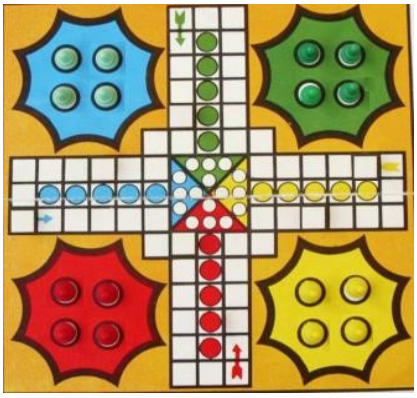
Second part of the lesson

- Hand out the worksheets one to each person
- Follow the instruction on the worksheet
- The participants must be able to count up to 4 and to follow the sequence of the numbers
- They must have the ability to compare the numbers from 1 to 6
- They must understand general mathematics – multiplication and division to understand probability.

USEFUL HINTS

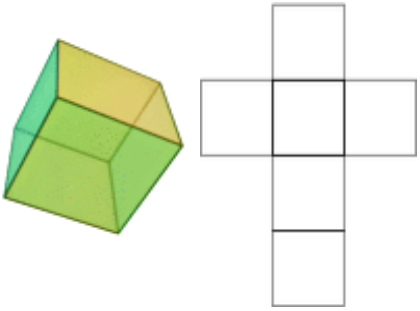
- At the end of the lesson, the worksheet should be completed.
- If the participants cannot read, the teacher has to guide them
- If participants have difficulties to count, you have to train – you need more time or you can split the group

WORKSHEET 2.2 PAGE 1 (LEARNER)



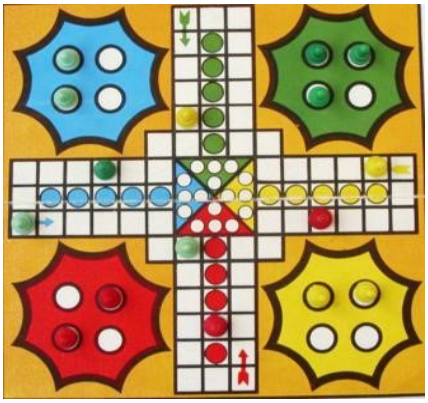
Rule: Each player takes four pawns. Each player counts and puts four figures in one colour into the small circles of the area ("yards") in the same colour on the board.

- How many pawns does each player have? _____
- How many participants are in the game? ___
- How many pawns are all together on the board? _____



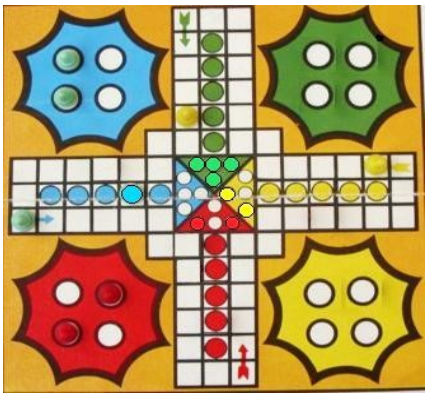
Rule: Each player rolls the dice; the player with the highest number begins the game. To enter a pawn into the playing field a player must roll number 6!

- What is the shape of the dice? _____
- How many sides does the dice have? _____
- Which numbers are symbolized on the sides of the dice? _____
- Which is the highest number _____
- Which is the lowest number? _____
- List the numbers written on each side of the dice as you start from the lowest to the highest _____



Rule: The player must always move a figure according to the dice value rolled. If no move is possible, pass the turn to the next player.

The game ends when all pawns of each player are in the final flight.

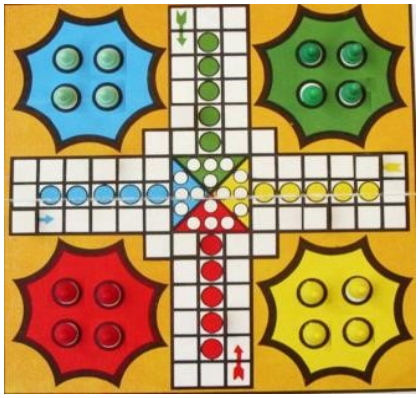


- How many fields must each player pass to move his pawn to the final (final flight)?

_____

_____

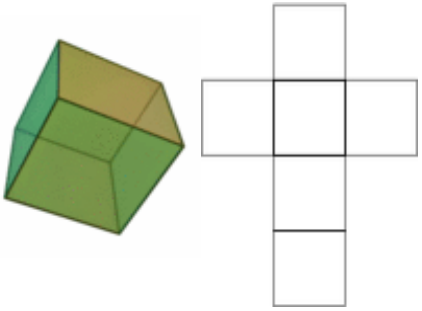
WORKSHEET 2.2 PAGE 1 (TEACHER)



Explain game rules to participants:

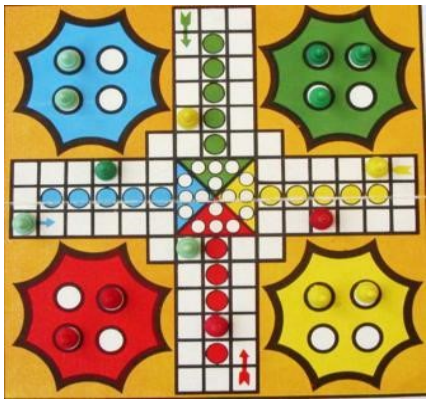
Rule: Each player takes four pawns. Each player counts and puts four figures in one colour into the small circles of the area ("yards") in the same colour on the board.

- How many pawns have each player? ***4***
- How many participants are in the game? ***4***
- How many pawns are all together on the board? ***4 + 4 + 4 + 4 = 16***

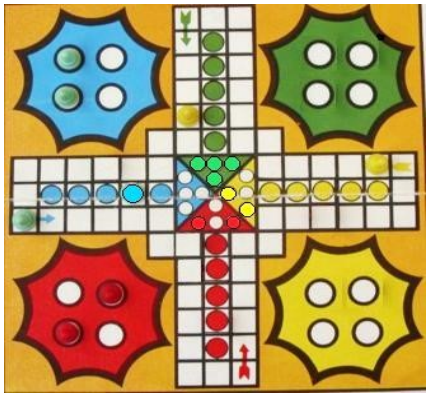


Rule: Each player rolls the dice; the player with the highest number begins the game. To enter a pawn into the playing field a player must roll number 6!

- What is the shape of the dice? ***cube***
- How many sides have the dice? ***6***
- Which numbers are symbolized on the sides of the dice? ***from 1 to 6***
- Which is the highest number ***6***
- Which is the lowest number? ***1***
- List the numbers written on each side of the dice as you start from the lowest to the highest ***1, 2, 3, 4, 5, 6***



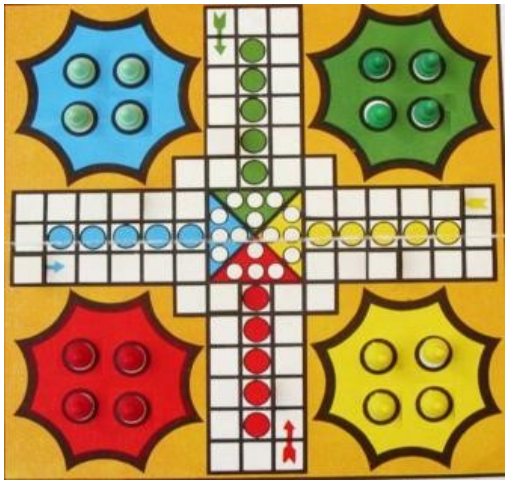
Rule: The player must always move a figure according to the dice value rolled. If no move is possible, pass the turn to the next player.



The game ends when all pawns of each player are in the final flight.

- How many fields must each player pass to move his pawn to the final (final flight)? ***one pawn 62, the second 63, the third 64 and the last one 65***

WORKSHEET 2.2 PAGE 2 (LEARNER)

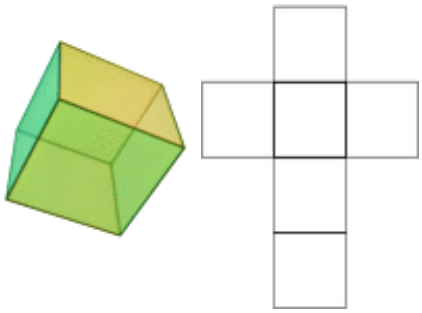


Task to learn probability

Roll the dice 100 times and mark with a | the results in a table, form the sum and divide by 100.

Number of the Dice	1	2	3	4	5	6
Frequency when rolling						
Sum						
Sum/100						

Result: In Statistic we say "The probability in the experiment is _____"



Question: What is the probability to get 3 when you roll the dice? _____

Using classical mathematical approach, the probability is $P(A) = \frac{m}{n}$

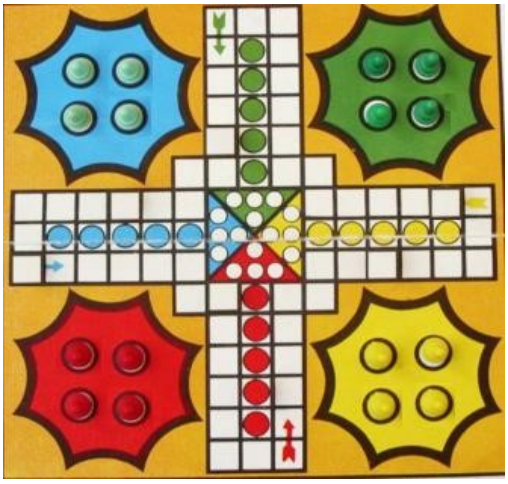
m is 1, as there is only one 3
and n is 6, as we have 6 sides of the dice.
If we assume that A is the probability to get 3,

P(A) = _____
This means, that the probability to roll a 3 is _____

Task:
Calculate the volume of a cube?
The formula for the Volume is
$V = a \cdot a \cdot a$
when "a" is the length of one of its sides.
If a=2 cm the Volume is
$V =$ _____

Task:
Calculate the surface area of the cube?
The Formula of the surface area is
$S = 6 \cdot a \cdot a$
"a" is the length of one of the 6 sides of the dice! If a=2 cm the surface is
$S =$ _____

WORKSHEET 2.2 PAGE 2 (TEACHER)

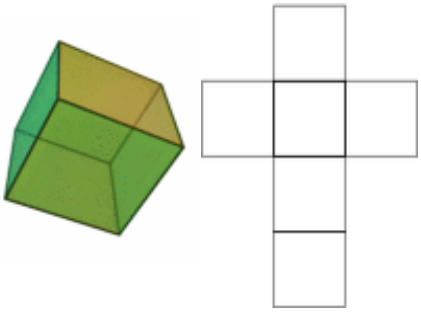


Task to learn probability

Roll the dice 100 times and mark with a | the results in a table, form the sum and divide by 100.

Number of the Dice	1	2	3	4	5	6
Frequency when rolling	||||| ||||| ||||| |	||||| ||||| ||||| |||	||||| ||||| ||||| ||||	||||| ||||| ||||| ||	||||| ||||| ||||	||||| ||||| ||||| |
Sum	16	18	19	17	14	16
Sum/100	0,16	0,18	0,19	0,17	0,14	0,16

Result: In Statistic we say "The probability in the experiment is ***0,16 to roll a 1 and 0,18 to roll a 2 ….***"



Question: What is the probability to get 3 when you roll the dice? ***0,19***

Using classical mathematical approach, the probability is $P(A)=\frac{m}{n}$ m is 1, as there is only one 3

and n is 6, as we have 6 sides of the dice.

If we assume that A is the probability to get 3,

P(A) = $\frac{1}{6}$ = ***0,166***

This means, that the probability to roll a 3 is ***1:6=0,166***

Task:

Calculate the volume of a cube?

The formula for the Volume is

$V = a \cdot a \cdot a$

when "a" is the length of one of its sides.

If a=2 cm the Volume is

$V = 2\ cm \cdot 2\ cm \cdot 2\ cm = 8\ cm^3$

Task:

Calculate the surface area of the cube?

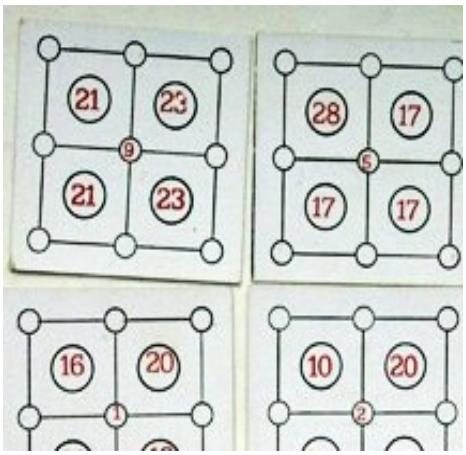
The Formula of the surface area is

$S = 6 \cdot a \cdot a$

"a" is the length of one of the 6 sides of the dice! If a=2 cm the surface is

$S = 6 \cdot 2\ cm \cdot 2\ cm = 24\ cm^2$

2.3 Combination Nine (Board Game)



Objectives

- The game was chosen, because of its brain drilling capabilities. It adheres to levels three “Applying” and six “Evaluating” of “Taxonomy of Educational Objectives”
- Participants should be taught to sum four single digit numbers fast and reliably
- They understand that if numbers are rearranged the sum is different
- They try to rearrange numbers until preliminary given result is reached

Tools, Materials and Organisation

- Game cards. You can make the cards by your own - see the template at the end of this lesson.
- Prepare copies from the worksheet for each student
- The lesson takes 45 minutes

Description of the Lesson

First part of the lesson

- The players are choosen.
- Explain the game “Combination 9”.
- The participants play several times the game “Combination 9” to learn the rules and objectives. On first tries no time limitation is defined.
- The participants learn to sum and rearrange numbers until the solution is found.
- Players raise hand when they find a solution.
- The others students can check their answer. After that teacher checks the correctness of the answer.
- When the participants become more experienced competition is announced. When participant finds a solution the time is recorded. The fastest wins.
- The game could be played in rounds of several games each.
- The competition may be for individuals or played by teams.
- Introduce the idea that the game has optimal strategy to find a solution. Remember that there are 362880 card variants including symmetric.

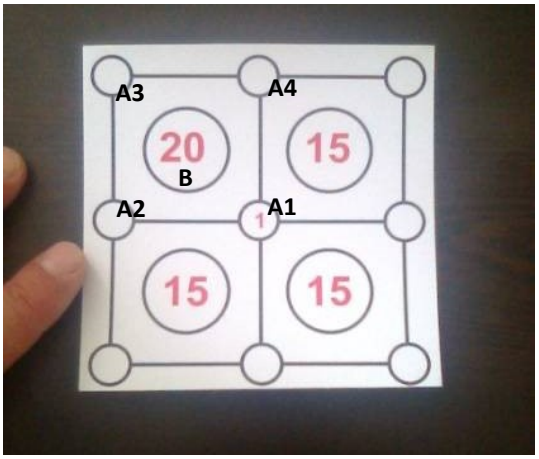
Second part of the lesson

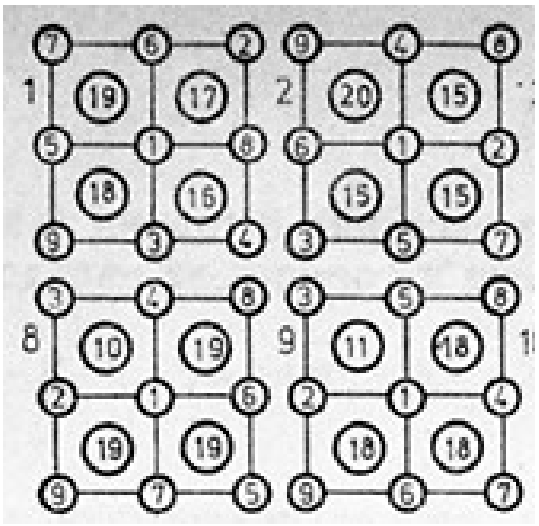
- Hand out the worksheets to each person one.
- Follow the instruction on the worksheet.

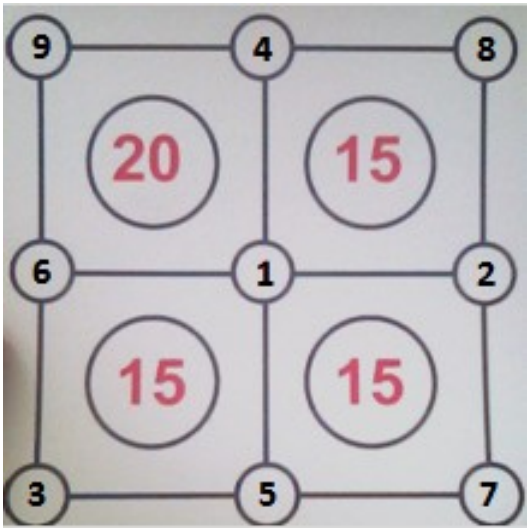
Useful Hints

- At the end of the lesson the worksheet is completed.
- If the participants cannot read, the teacher has to guide them.
- If the participants can’t understand the rules, the teacher has to guide them.
- If participants have difficulties to sum and combine numbers, you have to divide the lesson into two lessons: First lesson: sum numbers, second lesson: rearrange numbers.

WORKSHEET 2.3 (LEARNER)







Take the card.

Look at it.

Try to find a solution.

You can keep a draft of intermediate calculations.

Try different variants.

Try to use a strategy.

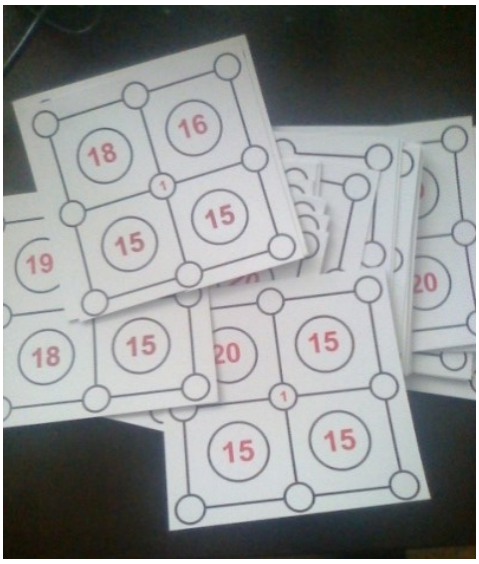
Question

If you have three numbers in the corner of the little square (A1=1, A2, A3) and the result in the middle of the box B: how can you find the number A4 in the fourth corner of the square?

When the solution is found present it to the teacher!

If you participate in a competition, raise a hand or stop the timer.

WORKSHEET 2.3 (TEACHER)



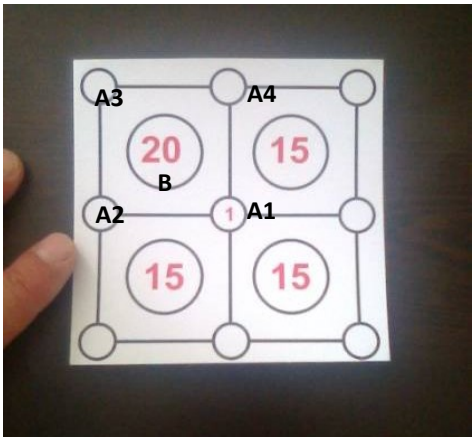
Explain game rules to participants

Start the game Combination 9 by shuffling cards.

Give each player a card.

The player takes the card and tries to find a solution.

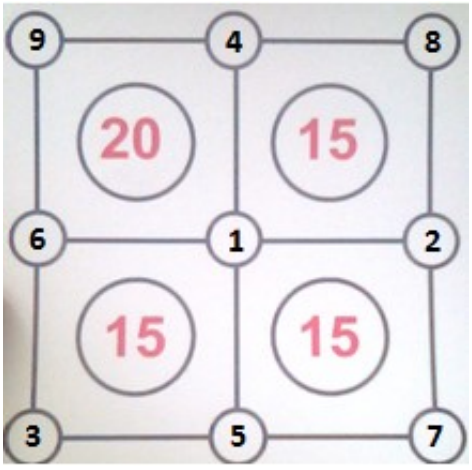
He can keep notes and drafts on additional notebook.



Question

If you have three numbers in the corner of the little square (A1=1, A2, A3) and result in the middle of the box B: how can you find the number A4 in the fourth corner of the square?

$$A4 = B - (A1 + A2 + A3)$$



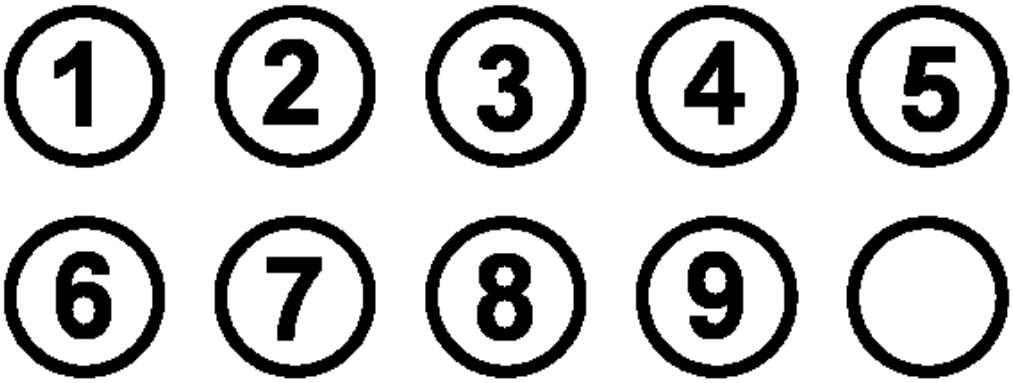
The player presents the solution to the other students and they check their answer.

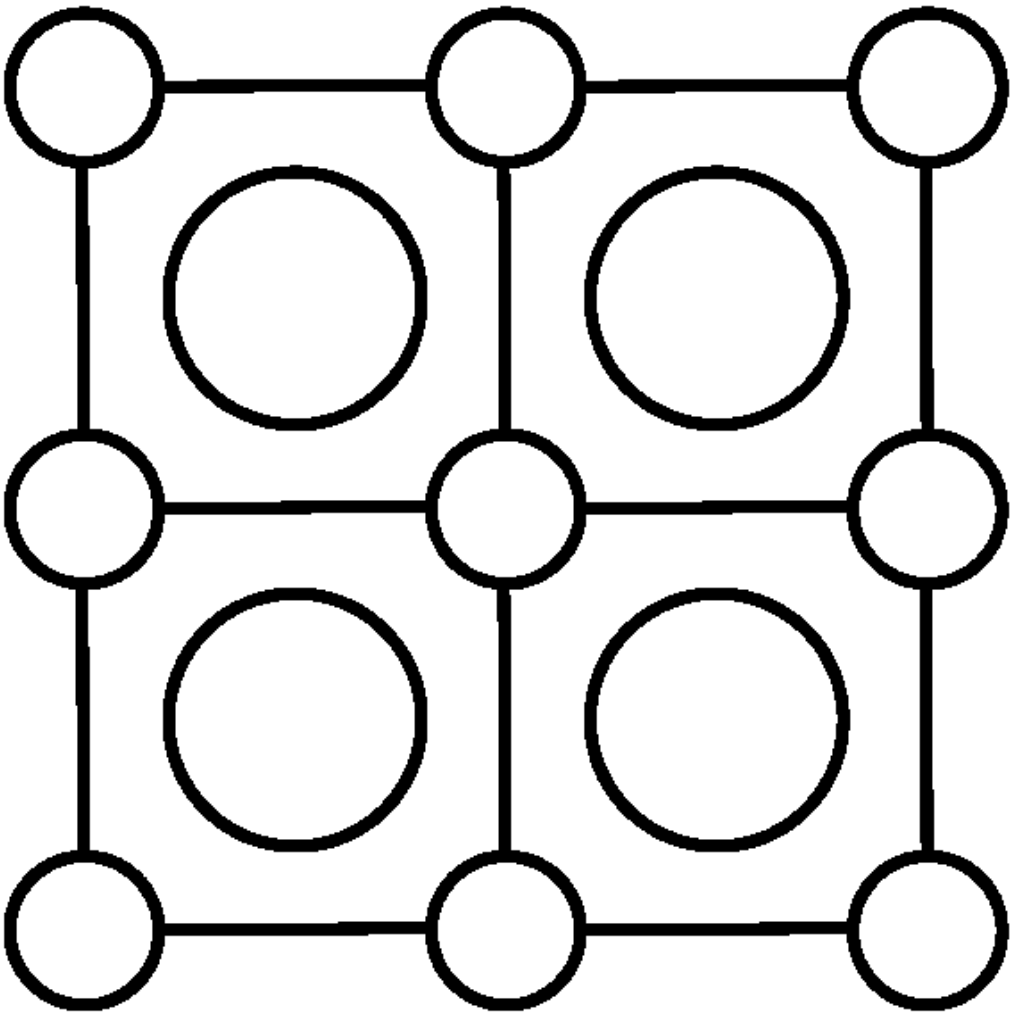
After that teacher checks the correctness of the answer

After the students feel confident with the game the teacher could start time-based competition.

TEMPLATE 2.3 TO MAKE YOUR OWN CARNETS "COMBINATION NINE"

Step 1: Put the numbers from 2 to 9 disordered into the small circles. 1 is always in the middle-circle.
Step 2: Add the numbers in the 4 small circles, which are around one bigger circle and write the sum in the big circle.
Step 3: Remove the 9 numbers from the small circles and your carnet is finished!





3.1 Black Jack (Card Game)

Objectives

- Count and add numbers of value 1-11 up to 21+
- Read numbers up to 11
- Order and compare sum of numbers up to 21
- Improve memory for number cards
- Understand a logical system of rules with numbers

Tools, Materials and Organisation

- One or several standard deck of cards, depending on the number of players
- A convenient table for players to use
- The game has no time limit or you said the rules when the game ends, see below

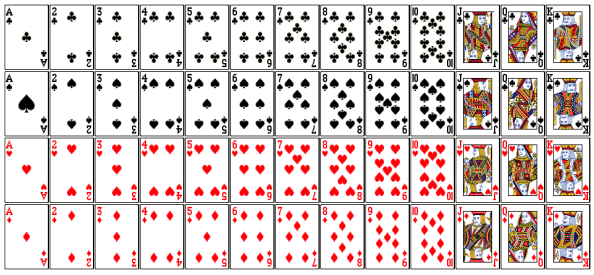
Description of the Lessons

Lesson 1:

- Identify the cards, type and value
- Understand what is a deck of cards
- Review the value of the cards
- Practice adding two cards each time and ask participants to give the value
- Practice adding more than 2 cards until the total is over 21
- Compare random pair of cards for their total value

Lesson 2:

- Explain the game “Black Jack”
- Arrange groups with at least 3 persons.
- Explain the role of a dealer or house. One of them has the role of the dealer or the teacher could have this role in the beginning.
- The participants play several rounds the game "Black-Jack" in order to understand the rules. The game is played with open cards for all, so in this way the learning of the rules becomes faster.
  The Players have fun in risking or in trying to remember the cards passed estimating chances/probabilities. Cards of value 10 have a higher chance to appear than any other number card.



- The participants learn to count up to 30+ by adding the value of the cards in their hand.

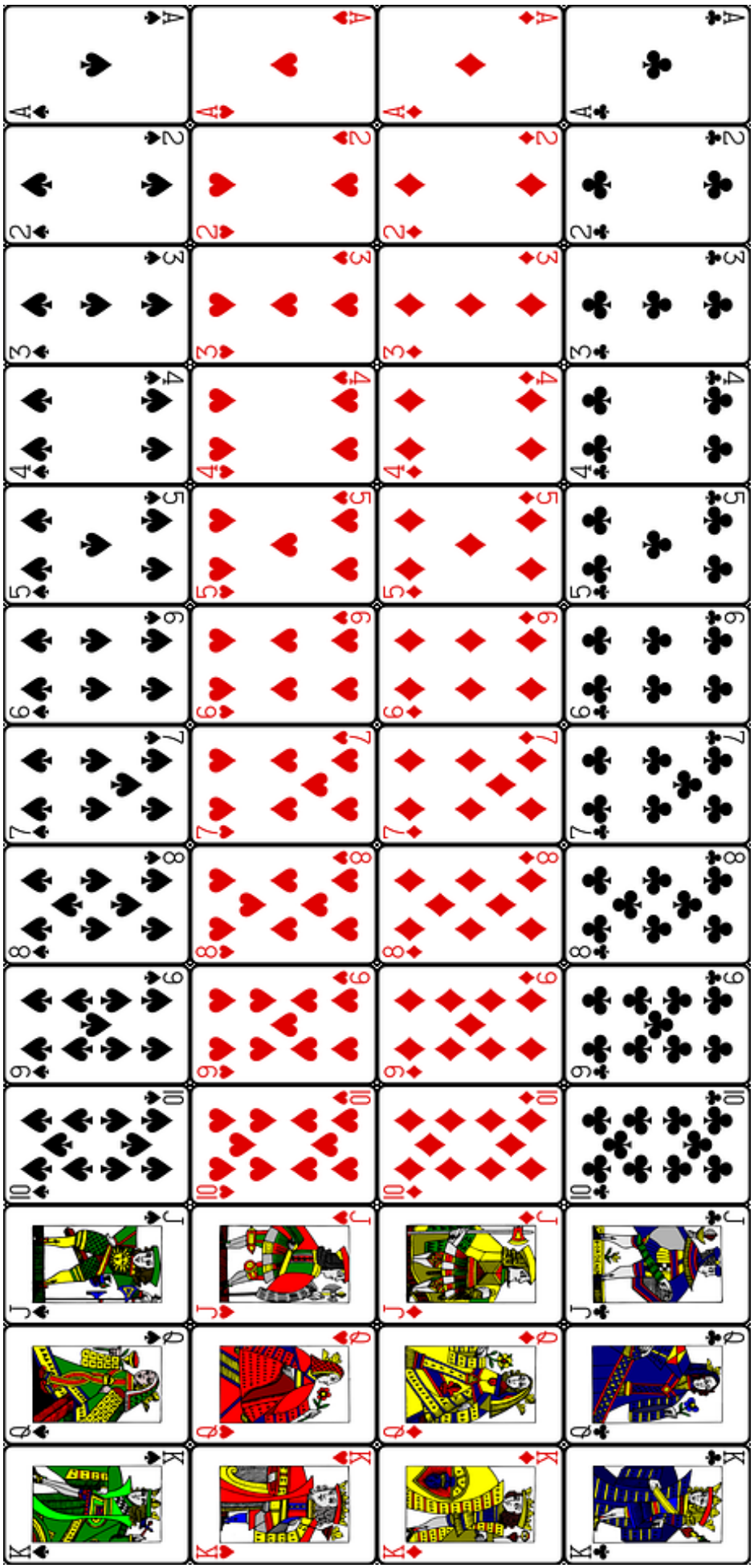
Lesson 3:

- Now the players play with closed cards to themselves.
- In order for the game to have an end each player receives 10 chips. When one of the players’ loses all the chips then the game can stop, so a winner will be the player with most chips left. In Casinos this is not the case as the game does not stop as long as there is at least one player. The game could decide to start with more chips, like 20 for example. The dealer should have at least as many starting chips of the players. If the dealer runs out of chips then again the game ends and the winner is the one with the highest number of chips.
- Additional features like
  Insurance, Surrender, Splitting, Doubling down

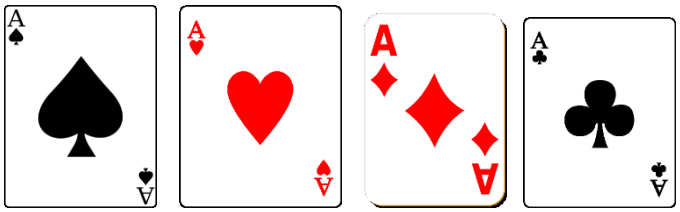
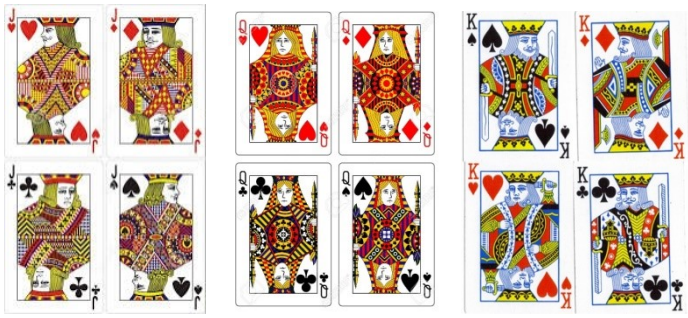
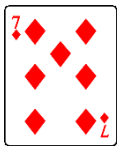
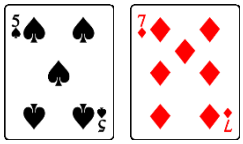
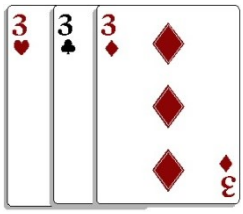
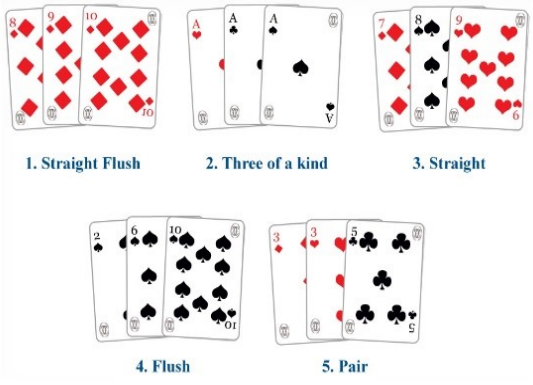
Useful Hints

- If the participants cannot add the value of the cards, the teacher or the dealer has to guide them.
- If participants have difficulties to count, you have to train – you need more time or you can do more open cards games.
  If it is easier for participants to write the number/value of the cards and then add them, provide paper and give them a chance to do it.

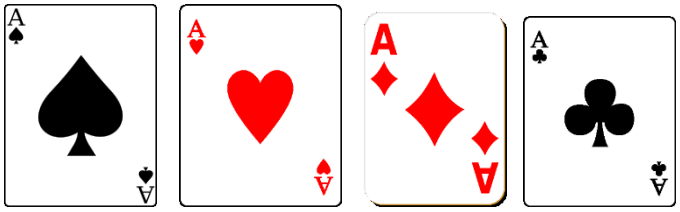
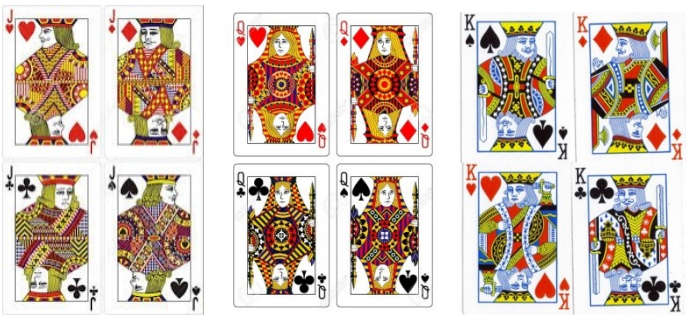
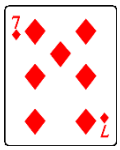
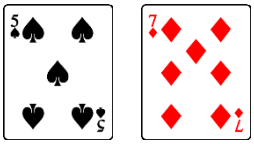
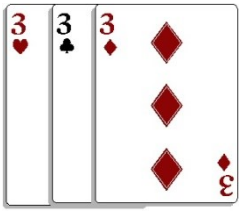
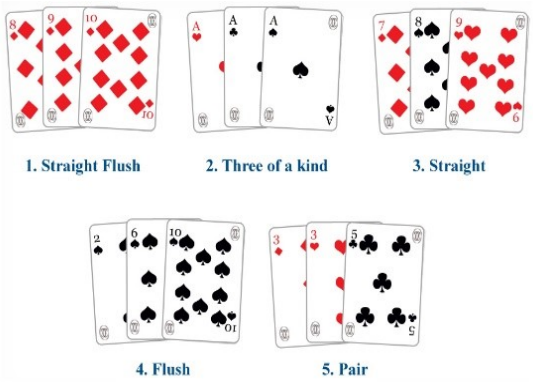
TEMPLATE



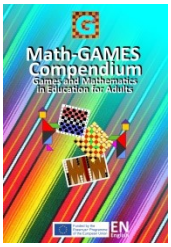
WORKSHEET 3.1.1 (LEARNER)

	Name the four suits: _____ _____ _____ _____
	What are the suit names of these cards? _____ _____ _____ _____ What is the value of each card? _____
	Name these face cards! _____ _____ _____ _____ What is the value of each? _____
	Name this card. _____ What is its value? _____
	What is the value of these two cards together? _____ + _____ = _____
	What is the value of these 3 cards together? ___ + ___ + ___ = _____
 1. Straight Flush 2. Three of a kind 3. Straight 4. Flush 5. Pair	Give the value of each of these hands: 1. _____ 2. _____ 3. _____ 4. _____ 5. _____

WORKSHEET 3.1.1 (TEACHER)

	Name the four suits: *Spade, Heart, Diamond, Club*
	What are the suit names of these cards? *Spade Ace Heart Ace* *Diamond Ace Club Ace* What is the value of each card? *1 or 11*
	Name these face cards! *Jack of Heart, Diamond, Club and Spade* *Queen of Heart, Diamond, Club and Spade* *King of Heart, Diamond, Club and Spade* What is the value of each? *10*
	Name this card. *Diamond Seven* What is its value? *7*
	What is the value of these two cards together? *5 + 7 = 12*
	What is the value of these 3 cards together? *3 + 3 + 3 = 9*
	Give the value of each of these hands: *1. Straight Flush 8+9+10=27* *2. Three of a Kind 11+11+11=33* *3. Straight 7+8+9=24* *4. Flush 2+6+10=18* *5. Pair and Five 3+3+5=11*

WORKSHEET 3.1.2 (TEACHER/LEARNER)



To answer these questions you should see Math-GAMES Compendium page 64 - 67!

You can download it from the website www.math-games.eu

What is a dealer or house?	
What is Black Jack? Show the hand of the Black Jack. What is its value?	
What does split pairs mean?	
What does Doubling down mean and when should one do it?	
What does surrender mean? And when do you do that? How much do you lose when you surrender?	
What is Insurance? When should a player buy insurance?	
„Black Jack pays 3 to 2“ is one of the important rules. • You bet 20€, how much will you get, if you win? • You bet 32€, how much will you get, if you win? • If you win and get from the dealer 6€, how much was your bet? • If you win and get from the dealer 60€, how much was your bet?	
The standard denominations for chips, also used in casinos are: white chips = €1, red chips = €5, green chips = €25, black chips = €100 • You bet by placing 2 white, 1 red and 2 green chips. How much did you bet? • You bet by placing 1 white, 2 red and 3 black chips. How much did you bet? • You bet with 12 €, which chips you have to give? • You bet with 64 €, which chips you have to give?	

3.2 MATH SCRABBLE (BOARD GAME)

OBJECTIVES

This game can be used to attain a broad range of objectives depending on the background of the learners the degree and the causes of their characterization as slow learners and all the related issues. Among these we identify the following ones as particularly attainable through this.

Objectives for Mathematical Content

C1. Recognize the meaning/ representation of the digits **0 1 2 3 4 5 6 7 8 9** and the symbols **+ - × ÷ = ( )**
C2. Recognize the meaning and representation of positive integers in the range 0 … 1000.
C3. Adding, subtracting, multiplying and dividing integers in the range 0 … 100.
C4. Using a calculator for the above operations
C5. Constructing/ Writing equalities, using the symbols that are part of the MathScrabble game
C6. Checking the validity of an equality
C7. Understand a coordinate system and identify positions on it

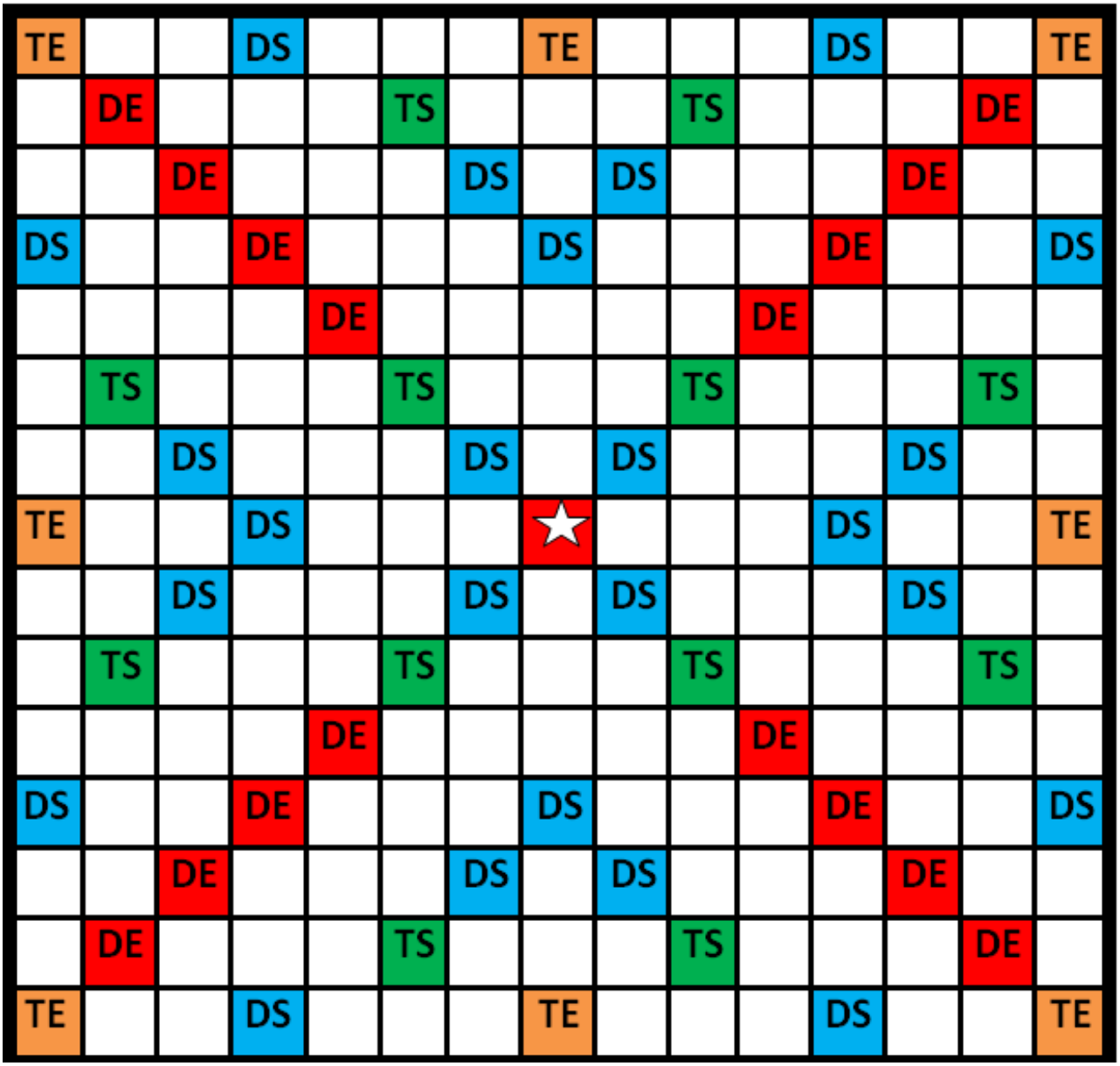
Objectives for the Development of General Mathematical Skills and Competencies

M1. Develop positive attitudes towards mathematics
M2. Build knowledge by taking advantage on interest and background experience of the learners
M3. Provide opportunities for exploring mathematical entities concepts and processes
M4. Encourage estimation skills
M5. View computation as a tool for problem solving and not as an end in itself
M6. Encourage multiple solutions strategies
M7. Develop students' calculator skills
M8. Provide opportunities for cooperation and group work
M8. Link numeracy and literacy skills
M9. Situate problem solving tasks in the context of real issues
M10. Develop skills for interpreting linguistic information and transferring it to numerical representation
M11. Develop problem solving skills (understanding a problem, devising a plan, implementing a plan, assessing the solution)
M12. Develop reasoning skills

TOOLS, MATERIALS AND ORGANISATION

In order to develop lessons through the exploitation of the Math Scrabble game it is expected that to use the Equipment for the game plus supporting material that provides supplementary illustrations that will help the students in developing their mathematical skills. This game material it is going to be needed:

THE BOARD FOR THE GAME



In addition and in order to facilitate the running of the game the following sheets are given aiming at helping the learners to keep track of the various activities: A card explaining the notation of the symbols on used on the board and a board with a system of coordinates.

You need tiles (see next page):
5×10 Tiles with numbers from 0 to 9
7×2 Tiles for Symbols + and –
5×22 Tiles for Symbols × and ÷
7×2 Tiles for Symbols (and)
20 Tiles with the Symbol =
4 Tiles are empty (Joker)

TE	**Triple the score corresponding to the equality(/ies) using such a cell**
DE	**Double the score corresponding to the equality(/ies) using such a cell**
TS	**Triple the score corresponding to the symbol using such a cell**
DS	**Double the score corresponding to the symbol using such a cell**
★	**Triple the score corresponding to the equality using such a cell**

KOPIERVORLAGE

0 Scoring 1	1 Scoring 1	2 Scoring 1	3 Scoring 2	4 Scoring 2	5 Scoring 3	6 Scoring 2	7 Scoring 4	8 Scoring 2	9 Scoring 2
0 Scoring 1	1 Scoring 1	2 Scoring 1	3 Scoring 2	4 Scoring 2	5 Scoring 3	6 Scoring 2	7 Scoring 4	8 Scoring 2	9 Scoring 2
0 Scoring 1	1 Scoring 1	2 Scoring 1	3 Scoring 2	4 Scoring 2	5 Scoring 3	6 Scoring 2	7 Scoring 4	8 Scoring 2	9 Scoring 2
0 Scoring 1	1 Scoring 1	2 Scoring 1	3 Scoring 2	4 Scoring 2	5 Scoring 3	6 Scoring 2	7 Scoring 4	8 Scoring 2	9 Scoring 2
0 Scoring 1	1 Scoring 1	2 Scoring 1	3 Scoring 2	4 Scoring 2	5 Scoring 3	6 Scoring 2	7 Scoring 4	8 Scoring 2	9 Scoring 2
= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2
= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2	= Scoring 2
+ Scoring 1	− Scoring 1	+ Scoring 1	− Scoring 1	+ Scoring 1	− Scoring 1	+ Scoring 1	− Scoring 1	+ Scoring 1	− Scoring 1
+ Scoring 1	− Scoring 1	+ Scoring 1	− Scoring 1						
× Scoring 2	÷ Scoring 3	× Scoring 2	÷ Scoring 3	× Scoring 2	÷ Scoring 3	× Scoring 2	÷ Scoring 3	× Scoring 2	÷ Scoring 3
Scoring 0	Scoring 0	Scoring 0	Scoring 0						
(Scoring 5	) Scoring 5	(Scoring 5	) Scoring 5	(Scoring 5	) Scoring 5	(Scoring 5	) Scoring 5	(Scoring 5	) Scoring 5
(Scoring 5	) Scoring 5	(Scoring 5	) Scoring 5						

Leerer Spielstein kann überall als Joker eingesetzt werden.

Information-Card 1: MATHSCRABBLE RULES - RULES FOR PLAYING

1. Each player takes initially 9 tiles from the bag.
2. Then each player is expected to construct, if possible, a valid equality using all or some of the tiles in his hand.
3. The first player that has a valid equality has to place it on the board by placing the symbol "=" in the central square (denoted by the star) and by arranging the other tiles either horizontally or vertically.
4. Equality can be read horizontally or vertically.
5. In each turn a **new (additional) symbol "="** can be used only once, that is if a player has in his/ hers hand two or more symbols "=" he/ she is allowed to use at most one of them for his/hers turn.
6. A player can construct a valid equality by constructing a new equality or by extending an existing one by using already positioned tiles on the board for developing one or more new ones , that is expressions with more than two equal parts (e.g. 1+1=2=5-3=8÷4)
7. Each player keeps always 9 tiles in his hand, thus after he/ she has constructed an equality he/ she picks up from the bag the same number of tiles as the ones he/ she has used for the construction. This requirement does not apply if there are no more tiles in the bag and in this case the player is left with less than 9 tiles.
8. The symbol "-" can be used either as the sign of a negative number or as the symbol of subtraction.
9. The game ends when either
   (a) There are no other tiles in the bag and the last player used all his tiles, or
   (b) There are no other tiles in the bag and no player can go out (i.e. can construct any valid equality and use all his tiles)

Information-Card 2: MATHSCRABBLE RULES -RULES FOR SCORING

For the scoring in each round

1. Find the total of the score by considering the point value of the tiles used for the construction of the present equality plus the extra points that can be gained from the consideration of the indications on the board in the squares that are used. The latter advantage (getting extra points as it is indicated on the squares of the board) counts only for the first time that a tile is placed on the board.
2. In case all nine tiles are used in the present round an extra bonus of 40 points is added for the score in the round.

For the scoring at the end of the game

Further to the total score of each player we have the following two cases depending on how the game ends:

1. In case (a) the score of the player that goes out is increased by adding the score values of the tiles with which the other players are left.
2. In case (b) the score of each player is decreased by the sum of the score values of the tiles with which he is left.

Sheets for Scoring in a Single round

Equality	
Score due to scoring value on the tiles used to construct the equality	
Score adjusted due to benefits of symbols, as specified by using squares marked DS the Board	
Score adjusted due to benefits of equalities as specified by using squares marked DE on the Board	
Score adjusted due to bonuses or penalties	
Total score for the round	

Sheets for keeping the records of scores for each player during the game

Round	**Player 1**	**Player 2**	**Player 3**	**Player 4**
Round 1				
Round 2				
…..				
Total Score				

MATH-SCRABBLE LESSON 3.2.1:

GETTING ACQUAINTED WITH BASIC MATHEMATICAL CONCEPTS USED IN MATHSCRABBLE

A lesson of 40 to 45 minutes duration

This lesson can be used as an Introduction to the ideas that concern the basic arithmetic symbols and other mathematical ideas as well as what they represent. Furthermore it provides the opportunity for developing skills for creativity and innovation.

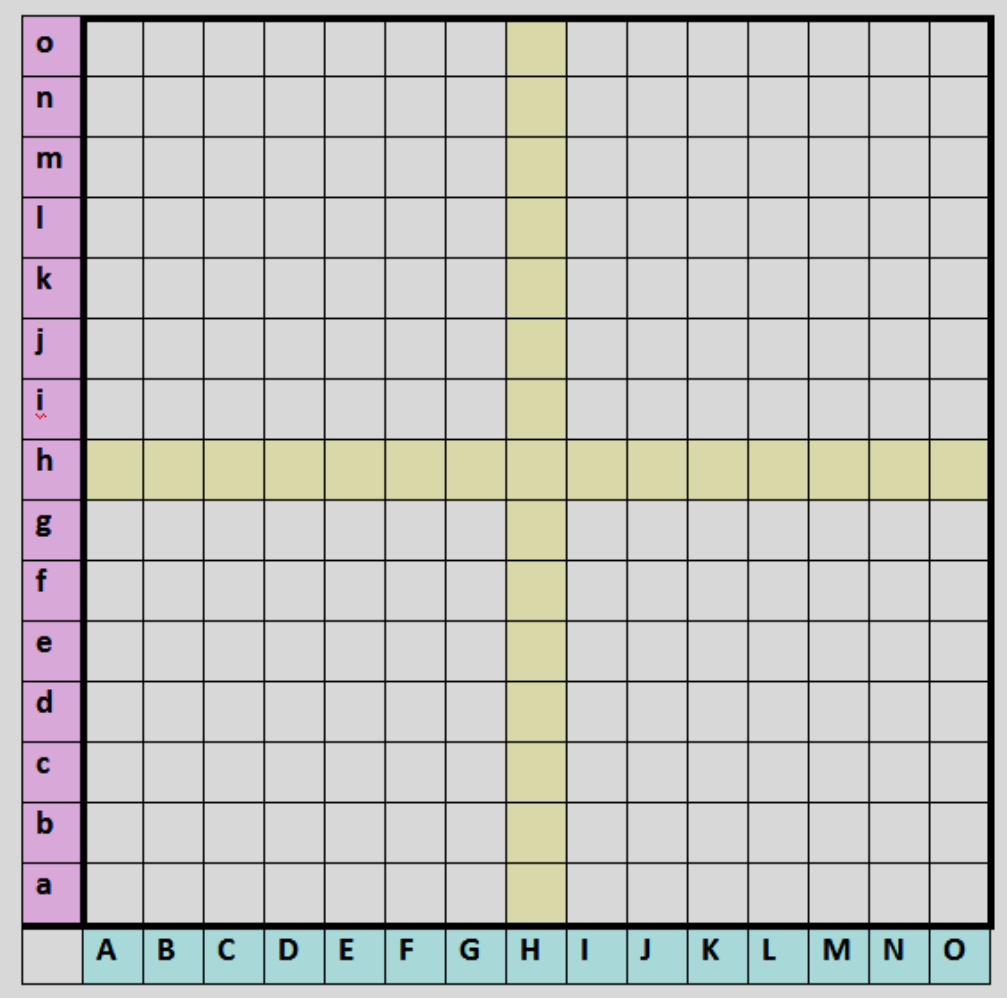
In particular this lesson aims at objectives C1, C2, C7, M1, M2, M3 and M11

Through this approach it is aimed to enable the students to recognize these symbols and to express what they stand for. In order to achieve this it is suggested to show them the basic tools of the MathScrabble game and ask them to construct them and explain what these represent or how they can be used in a mathematical context. Knowing the importance of their children to the majority of adults it is suggested that we can ask them to construct the various

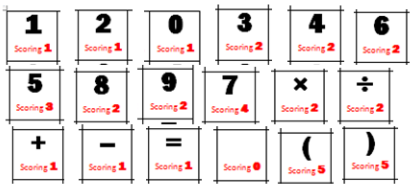
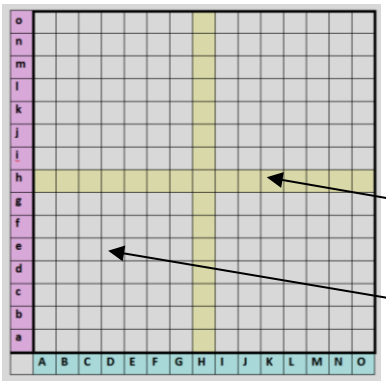
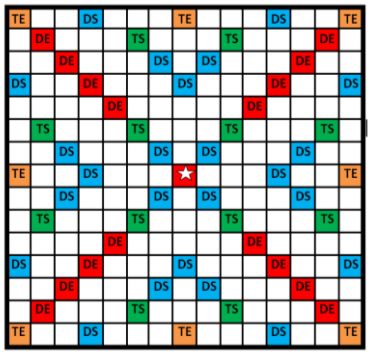
Tools that are needed in playing the game, by stressing that this game can be of help to their children for learning mathematics. This can prove to be an additional motivating factor.

In this lesson it is proposed

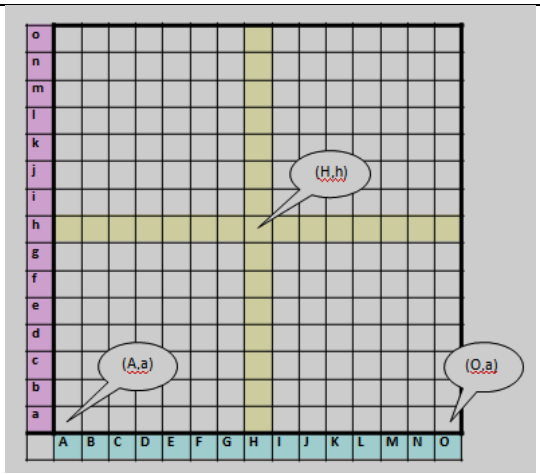
(a) to present to the students the tools and other material that are used in the MathScrabble
(b) to explain the meaning of these tools
(c) to present the rules of the game
(d) to help the learner in constructing and innovating



WORKSHEET 3.2.1 (LEARNER)

Information	**Requirements/ Questions for practice/ Comments**
Given the following tiles  	Name in your language and Explain the meaning of each of them. What is the value for the scoring on each of them? Which of them are arithmetical digits? Which of them are symbols for an arithmetic operation? Which symbol represents the equality of various quantities? What is the role of the "blank" symbol?
Consider the Board in its marked form that helps to identify the positions of the cells:  	Mark, using a red pencil, the positions of the following cells: (A,a), (B,c), (H,h), (M,c), (D,g) What do we mean by the coordinates of a cell? Find the coordinates of the following cells: The cell in the column marked by K and the row marked by e The cell in the row marked by a and the column marked by G The cell that is determined by the arrows: Arrow 1: ( ) Arrow 2: ( )
Consider the Board in its actual form: 	What happens if a tile is placed in the following cells (provided that it is valid to place it there): (Scoring 5 in (A, a) -> _____ Scoring in (D, d) -> _____ = Scoring 1 in (F, b) -> _____ ÷ Scoring 2 in (H, h) -> _____ 8 Scoring 2 in (L, o) -> _____
Given the cards with the rules for playing the game	Read the rules and reflect on these rules. Which terms/ concepts do you feel that you know and which you consider as not clear Discuss these ideas between you
Given cardboard, a pair of scissors, colored pencils, Geometrical instruments	Construct the various tools we need for playing MathScrabble What other material could you think of using for the construction of the Board, the tiles and whatever else we need? Can you think of more sophisticated constructions? Can you cooperate in achieving such aims? Discuss these ideas between you

WORKSHEET 3.2.1 (TEACHER)

Questions/ issues for discussion/ reflection	**Comments/ remarks**
What are the tools we need for playing MathScrabble? Can you explain how do you plan to introduce these tools to the learners?	Obviously the teacher should be aware of the tools and other supporting material for the game
How do you identify probable weaknesses of the learners concerning the meaning / representation of the symbols used on the tiles and the form of the Board? Can you take up the opportunity for helping them to consolidate these representations?	As the range of reasons for the weaknesses is quite broad it is important to develop some tools for identifying them and adapt the approach. For example if the learners are immigrants with very vague knowledge of the language the teacher should use appropriate approaches for explanations.
How do we explain the coordinating system that can be used for accessing the various cells on the Board? Can you develop examples/ exercises for it?	
How do you help the learners to comprehend the rules for the game?	One of the main problems we face in the learning process is the overcoming of the difficulties for reading and understanding.
How do you help the learners in constructing the basic tools and other supporting material? Can you develop a set of instructions for this?	By challenging the learners to construct we achieve a fruitful, pleasant and effective learning
Can you think of other issues for discussion/ reflection in order to achieve the objectives C1, C2 M1, M2, M3?	
Can you develop worksheets for the learners (in the spirit of the ones that follow)?	These might be similar or extensions of the ones that follow, but also might be altogether different aiming at either extending the ideas achieving the aims of the lesson or attending particular difficulties of the learners, depending on the reasons for their slowness.

MATH-SCRABBLE LESSON 3.2.2:

CONSTRUCTING EQUALITIES USING THE MATHSCRABBLE SYMBOLS

A lesson of 40 to 45 minutes duration

This lesson can be used as an opportunity for consolidation of the basic arithmetic operations and the idea of equality as a relation connecting equal quantities. Furthermore it provides the opportunity for developing problem solving and critical thinking skills. In particular this lesson aims at objectives C1, C2, C3, C4, C5, C6, M1, M2, M3, M6, M7, M8, M11, M12, M13.

Through this approach it is aimed to enable the students to recognize the meaning of equality and to provide them with opportunities for identifying valid ones. Furthermore it is expected to provide opportunities for constructing ones themselves using as many tiles as possible.

In this lesson it is proposed

(a) to present to the students the idea of an equality
(b) to provide opportunities for basic arithmetic operations
(c) to understand the process of simple problem solving

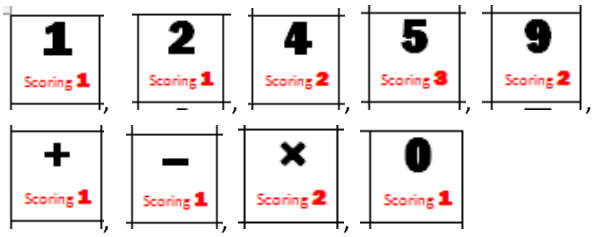
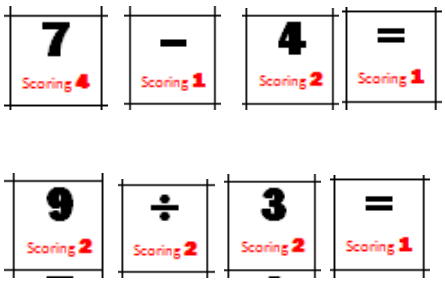
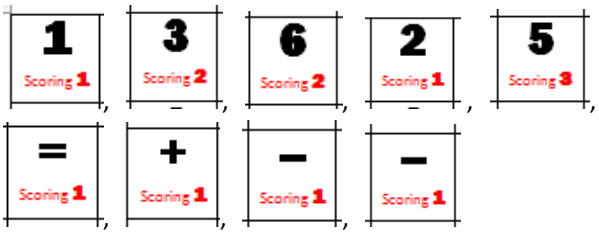
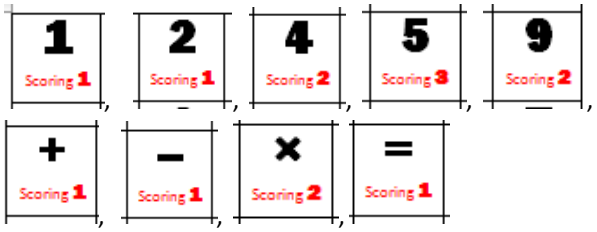
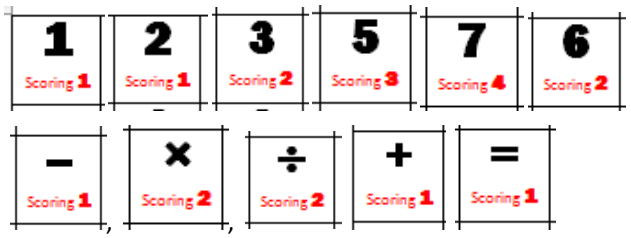
In solving the simple problem of constructing equalities, it is useful, to follow the next process:

- Do we understand the problem? (What are the data, what is required; do we know what are the meaning /role of the various terms involved?)
- Can we devise a plan of how to work? (can we construct quantities (in the form of a mathematical expression) on two sides using the tiles and calculate the outcome for each side)
- Can we implement our plan? (by putting down the various expressions and doing the calculations for each side) and give an answer
- Can we investigate the correctness of our answer? (Is our answer valid? Is this the only answer? Is this the best answer?)

WORKSHEET 3.2.2 (TEACHER)

Questions/ issues for discussion/ reflection	**Comments/ remarks**
How do we consider the issue of quantity? How do we explain the concept of equality?	 Does the traditional weighing scale provide a helpful approach?
Provide sets of tiles aiming at using them for constructing quantities (using basic arithmetic operations) and calculating the outcome	
Provide groups of equalities and ask to identify the valid ones	It is an opportunity for discussion of what is involved in checking the validity of an equation
Provide sets of tiles and ask the learner to construct equalities	It is an opportunity for discussion of the process of problem solving
Develop worksheets for the learners aiming at calculating quantities and constructing valid equalities, using the tools of the MathScrabble.	The examples that follow are indicative but obviously you can develop many other worksheets
Solution of the last question page 64	2*5-1=6/3+7

WORKSHEET 3.2.2 (LEARNER)

Information	**Requirements/ Questions for practice/ Comments**
Given are the following expressions (equations): a. 5+3 = 8 b. 8-2 = 2×3 c. 4÷2 = 2+0 d. 12 = 6+2 e. 7 - 2×2 = 10 - 7 f. 9 - 6÷3 = 5+2	Which expression/quantity is the left side and which one is the right side? What is the outcome of the each expression (left or right side) for each relation? **These equations are all valid expressions! In the game you always use valid expressions!**
Given the following tiles 	Find the outcome for each of the following expressions using tiles from the given set to present your answer 
Given the following nine tiles 	Which of the following relations can be constructed using these tiles and which of them are valid ones? (a) $5+1=6$ (b) $6-2=5-1$ (c) $6\div 2=3$ (d) $3+1=6-2$ (e) $3\times 2=6$ (f) $5-2=3$ (g) $6-5=3-2$ (h) $5-1=6+2-3$
Given the following nine tiles 	Construct valid equalities using **some** of these tiles! Find three different equations! Calculate the score value of these equations!
Given the following nine tiles 	Construct valid equalities using **all** of these tiles! (Solution see teacher's part) Calculate the score value of the equation!

MATH-SCRABBLE LESSON 3.2.3:

CONSTRUCTING EQUALITIES AND PLACING THEM ON THE BOARD USING THE SYMBOLS

A lesson of 40 to 45 minutes duration

This lesson can be used as an opportunity for consolidation of the basic arithmetic operations and the idea of equality as a relation connecting equal quantities. Furthermore it provides the opportunity for developing problem solving and critical thinking skills. In particular this lesson aims at objectives C1, C2, C3, C4, C5, C6, M1, M2, M3, M6, M7, M8, M11, M12, M13. The additional effort for this lesson is that the learner will have to utilize/ exploit already existing equalities on the Board by adding his/ hers tiles and create new ones.

Through this approach it is aimed to enable the students to recognize the meaning of equality and to provide them with opportunities for identifying valid ones. Furthermore it is expected to provide opportunities for constructing ones themselves using as many tiles as possible.

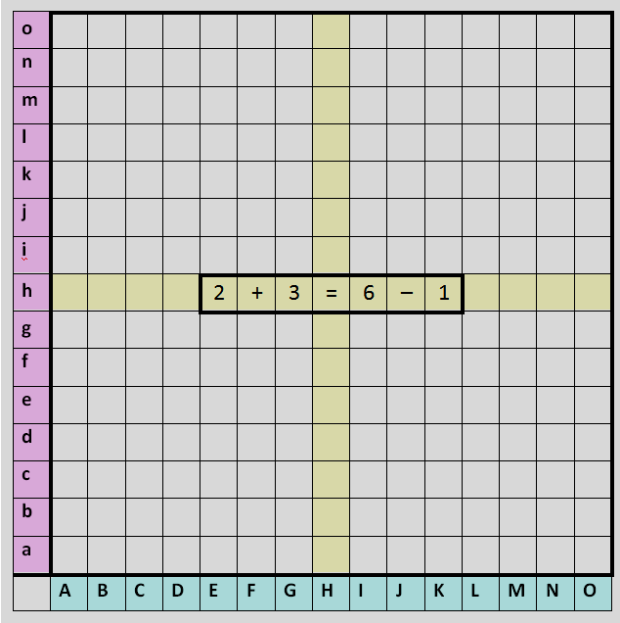
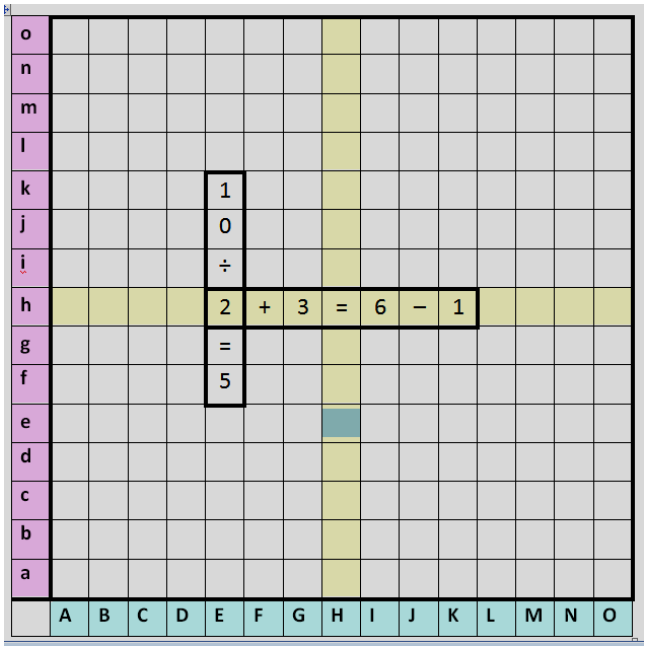
In this lesson it is proposed

(a) to provide opportunities for basic arithmetic operations and for using the symbols included in the tools of the game
(b) to place tiles on the Board so that they represent valid equalities and so that they are making use of already existing ones
(c) to oblige them to check the work of their competitors and to exploit the various developments in order to achieve the best for themselves
(d) to understand the process of simple problem solving

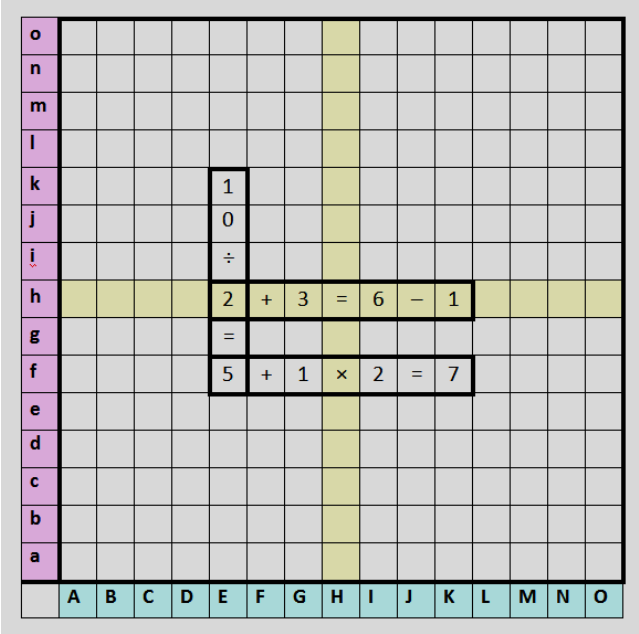
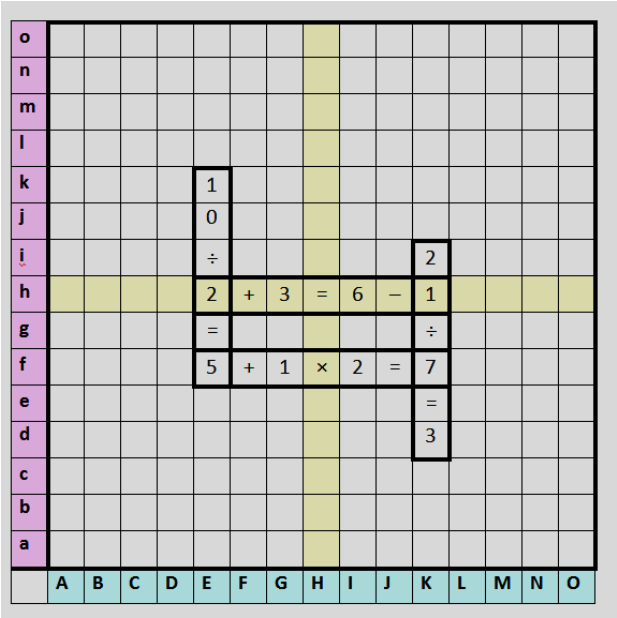
WORKSHEET 3.2.3 (TEACHER)

Questions/ issues for discussion/ reflection	**Comments/ remarks**
What are some important points that have to be taken into consideration when a learner has to place his/ hers equality on the Board?	Consider tiles on the Board and what are in player's hands Construct various equalities using all the tiles Work vertically and horizontally Have in mind the score (see next lesson) Distinguish the approach of the first round from the others The need to play so that the player managing the best for themselves and also hindering the next players.
How do we communicate with the learners in order to understand the positions (coordinates) for placing their tiles How do we challenge them in order to achieve solutions with the various advantages?	
Construct worksheets for the learners so that they are given tiles in succession and they are asked to place them on the Board according to the rules of the game	The example in sheet 8 is indicative of the process

WORKSHEET 3.2.3 PAGE 1 (LEARNER)

Information	**Requirements/ Questions for practice/ Comments**
The following tiles are in the hands of Player 1 1, 2, 3, 4, 6, +, – , =, ÷	Construct an equality and place it on the Board
An answer given to the previous question is the following 	Check whether this is a valid equality and whether it abides by the rules of the game. Explain your position At what coordinates is the symbol "=" placed?
The following tiles are in the hands of Player 2 when asked to go for the second round of the game 0, 1, 3, 4, 5,9, +, – , =, ÷	Construct an equality and place it on the Board
An answer given to the previous question is the following 	Check whether this is a valid equality and whether it abides by the rules of the game. Explain your position At what coordinates is the symbol "=" placed? How many of the tiles in his hand does he use?

WORKSHEET 3.2.3 PAGE 2 (LEARNER)

The following tiles are in the hands of Player 3 when asked to go for the third round of the game 0, 1, 3, 4, 5,7, +, – , =, ×	Construct an equality and place it on the Board
An answer given to the previous question is the following  	Check whether this is a valid equality and whether it abides by the rules of the game. Explain your position At what coordinates is the symbol “=” placed? How many of the tiles in his hand does he use?
The following tiles are in the hands of Player 4 when asked to go for the fourth round of the game 0, 1, 2, 3, 7, +, – , =, =, ×	Construct an equality and place it on the Board
An answer given to the previous question is the following  	Check whether this is a valid equality and whether it abides by the rules of the game. Explain your position At what coordinates is the symbol “=” placed? How many of the tiles in his hand does he use?

3.3 MONOPOLY (BOARD GAME)

OBJECTIVES

This game can be used to attain a broad range of objectives depending on the background of the learners the degree and the causes of their characterization as slow learners and all the related issues. It is a game that is immediately connected with the everyday applications of mathematics and particularly the ones that relate with money. As its name (Monopoly) stresses it is a game that deals with selling and buying, an activity that makes the learning of mathematics a necessity for everybody. The game concerns the dealings with property and it can be a strong motive for an adult to understand the process and the main goal which is to give the opportunity to the players to become wealthier (actually the winner of the game is the one that manages to have the assets with the highest value). The game is even useful for the people with a socialist philosophy, as by learning it they can understand the economic processes in the market and promote some actions for alleviating them (If you want to fight something you have to know it and understand it).
Among the objectives that can be promoted with this game we identify the following ones as particularly attainable through this.

Objectives for Mathematical Content

C1. Recognize the meaning/ representation of the digits **0, 1, …, 9** and the symbols **+ - × ÷ = ( )**
C2. Recognize the meaning and representation of positive integers in the range 0, …, 1000000
C3. Adding, subtracting, and multiplying integers in the range 0, …, 1000000
C4. Using a calculator for the above operations
C5. Understanding the fact that there is a relation of order in the set of integers and recognize how to compare such numbers
C6. Use the symbols <, >, and checking the order of integers

Objectives for the Development of General Mathematical Skills and Competencies

M1. Develop positive attitudes towards mathematics
M2. Build knowledge by taking advantage on interest and background experience of the learners
M3. Provide opportunities for exploring mathematical entities concepts and processes
M4. Encourage estimation skills
M5. Develop communication skills, using mathematical entities
M6. View computation as a tool for problem solving and not as an end in itself
M7. Encourage multiple solutions strategies
M8. Develop students' calculator skills
M9. Provide opportunities for cooperation and group work
M10. Link numeracy and literacy skills
M11. Situate problem solving tasks in the context of real issues
M12. Develop skills for interpreting linguistic information and transferring it to numerical representation
M13. Develop problem solving skills (understanding a problem, devising a plan, implementing a plan, assessing the solution)
M14. Develop reasoning skills

TOOLS, MATERIALS AND ORGANISATION

In order to develop lessons through the exploitation of the Monopoly game it is expected to use the Equipment for the game plus supporting material that provides supplementary illustrations that will help the students in developing their mathematical skills. This material is presented in the ADENDUM for the Section 3.3 MONOPOLY of the Guidebook. It is expected that the users of the game should have to buy a set, or more if needed, for playing the game, from a commercial shop. The game is available in the majority of the bookstores or big stationary shops, or even supermarkets for a reasonable price. The game has been translated in the majority of the European Languages as well as in many other languages.

Alternatively the material needed can be constructed by the learners with the additional value of giving them the opportunity for creation and comprehension of the various ideas involved

LESSON 3.3.1:

GETTING ACQUAINTED WITH BASIC MATHEMATICAL CONCEPTS USED IN MONOPOLY

A lesson of 40 to 45 minutes duration

This lesson can be used as an **Introduction** to the ideas that concern the basic arithmetic symbols and other mathematical ideas as well as what they represent. Particularly it can provide the opportunity for comparing two or more quantities and use them in every day transaction, thus providing the need for learning mathematics and realizing that it is an entity with broad **applications** in everyday life. Furthermore it provides the opportunity for developing skills for **creativity and innovation**.

In particular this lesson aims at objectives C1, C2, C5, C6, M1, M2, M3, M5, M11

Through this approach it is aimed to enable the students to recognize what numbers stand for and connect them immediately with values – money.

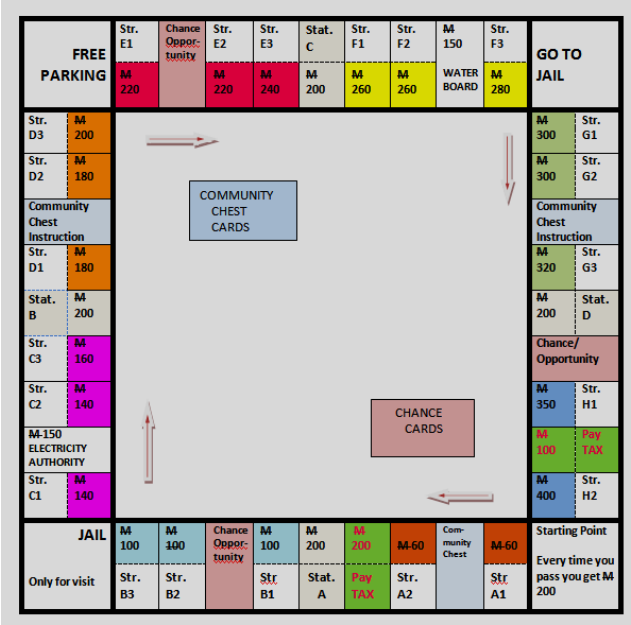
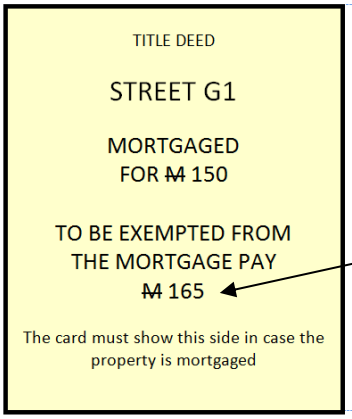
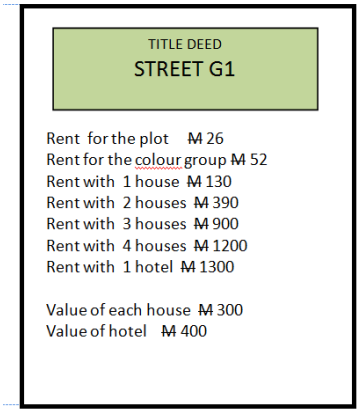
In this lesson it is proposed

- to present to the students the tools and other material that are used in Monopoly and identify the connection with real life
- to demonstrate the need for understanding mathematical concepts as a basic instrument in everyday life transactions
- to help the learner in constructing and innovating.

WORKSHEET 3.3.1 (TEACHER)

Questions/ issues for discussion/ reflection	**Comments/ remarks**
What are the tools we need for playing Monopoly? Can you explain how do you plan to introduce these tools to the learners?	Obviously the teacher should be aware of the tools and other supporting material for the game. Use the ADDENDUM Tools for Monopoly Present to the students the various materials for the game as can be acquired/ supplied from a commercial shop
How do you identify probable weaknesses of the learners concerning the meaning of the mathematical concepts that can be found on the Board as well as on the various cards Can you take up the opportunity for helping them to consolidate these representations?	As the range of reasons for the weaknesses is quite broad it is important to develop some tools for identifying them and adapt the approach. For example if the learners are immigrants with very vague knowledge of the language the teacher should use appropriate approaches for explanations
How do you help the learners to comprehend the rules for the game?	One of the main problems we face in the learning process is the overcoming of the difficulties for reading and understanding.
How do you help the learners in constructing the basic tools and other supporting material? Can you develop a set of instructions for this?	By challenging the learners to construct we achieve a fruitful, pleasant and effective learning
Can you think of other issues for discussion/ reflection in order to achieve the objectives C1, C2, C5, C6, M1, M2, M3, M5, M11	
Can you develop worksheets for the learners (in the spirit of the ones that follow)?	These might be similar or extensions of the ones that follow, but also might be altogether different aiming at either extending the ideas achieving the aims of the lesson or attending particular difficulties of the learners, depending on the reasons for their slowness.

WORKSHEET 3.3.1 PAGE 1 (LEARNER)

Information	Requirements/ Questions for practice/ Comments
Given the Board for the game (Provided that this is very small representation use the Board of the game that you have in your class or at home)  	Go over the various cells and identify on each of them the various stated numbers. Explain what these numbers represent. At what cells do you expect to have the opportunity of receiving money, how much and under what conditions? At what cells do you expect to have the obligation to give money either to the Bank or to other players? Identify on the Board the value/ price for each of the plots, by specifying the colour group and the street it lays, the railway stations and the public services. Explain what this price represents. Find the colour of the most expensive plots using the price on the appropriate cells. Can you identify the street of the most expensive plot (obviously at the initial stages of the game, as later it can be sold or bought at different prices)?
Consider the Title deeds for the various properties (totally 28) and go over the information written on them e.g. titles like the following: Front side of each card: The reverse side of each card:   	What does the information on these cards mean? When do we have pay rent and to whom? What is the highest price and what is the lowest price one has to use for renting a house in the case that there is at most one house in every plot? What is the name of the street where this occurs? What is meant by “mortgaging a property”, why do we have to do this (what do we get and from whom)? In order to be exempted from mortgage what do we have to pay? **ADVANCED QUESTIIONS** Given that the interest to be paid to the Bank for exempting from a mortgage is 10% find how much do you have to pay for the following mortgages?: (a) M 100 (b) M 150 (c) M 200 (d) M 80 (e) M 350 (f) M 120 (g) M 260

LESSON 3.3.2: USING MONEY FOR BUYING AND SELLING IN THE MONOPOLY GAME

A lesson of 40 to 45 minutes duration

This lesson can be used as a **consolidation** of using integers for simple arithmetic operations. Particularly it can provide the opportunity for using money for buying and selling as well as how to find the change to be given if the denominations of money at the disposal of a player do not allow direct payments. It can be used as a demonstration of how to handle money and use them in every day transaction, thus providing the need for learning mathematics and realizing that it is an entity with broad **applications** in everyday life. Furthermore it provides the opportunity for developing skills for **creativity and innovation**. Calculators can be allowed. In particular this lesson aims at objectives C1, C2, C3, C4, C5, C6, M1, M2, M3, M4, M5, M6, M8, M9, M10, M11, M12, M13, M14.

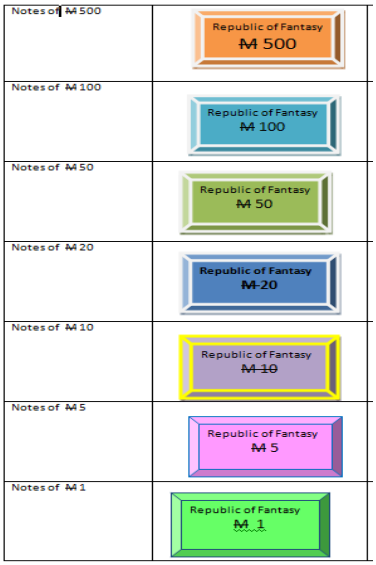
Through this approach it is aimed to enable the students to recognize what the numbers on banknotes stand for and How to use them in everyday transactions.

In this lesson it is proposed to provide the students with banknotes and ask them to use them for selling, buying and paying or receiving money for various activities (taxes, penalties etc.) and to demonstrate skills for handling money

WORKSHEET 3.3.2 (TEACHER)

Questions/ issues for discussion/ reflection	**Comments/ remarks**
As money is given in different denominations the learner should develop skills in handling them properly. Can you explain/ present ideas of how do you plan to introduce them to the learners?	Basic ideas should include: Recognition of the value/ denomination on each banknote Using the appropriate banknotes for buying, selling or doing other transactions. Skills for identifying the amount of change that has to be given in case of lack of the possibility to construct the exact amount with the existing banknotes.
Construct worksheets for the learners in order to practice these ideas	The example for Worksheet 5 is indicative

WORKSHEET 3.3.2 (LEARNER)

Information	**Tasks**
The transactions can be put into practice by using bank notes that are available in the following denominations: 	1. Given that you have the following banknotes: 2 of M 500, 3 of M 100, 2 of M 50, 3 of M 20 3 of M 10, 1 if M 5 and 5 of M 1. (a) Find what is the total amount in your possession, (b) Find what bills and how many of each kind are you going to use in order to pay the following amounts of money: M 200, M 70, M 650, M 24, M 163 (c) If you are given 3 bills of M 100, what is the total amount in your possession and how many bills do you have for each denomination? 2. Given that you have the following banknotes: 2 of M 500, 3 of M 100, 2 of M 50, 3 of M 20 3 of M 10, 1 if M 5 and 5 of M 1 and you arrive at the cell in Street A1, which is still unoccupied. How do you pay the bank in order to buy the plot and get the appropriate deed? 3. Given that you have the following banknotes: 2 of M 500, 1 of M 100, 2 of M 50, 3 of M 20 3 of M 10, 1 if M 5 and 5 of M 1. You want to pay M 400. How can you do that using the bills at your disposal? What is the amount of change you are going to receive for the case you are proposing? In what denominations of bills can this be achieved?

LESSON 3.3.3: KEEPING RECORDS OF THE ASSETS IN A PLAYER'S POSSESSION AND DOING THE NECESSARY CALCULATIONS FOR IT.

A lesson of 40 to 45 minutes duration

This lesson can be used as Lesson 2 but furthermore it can provide the opportunity for more elaborate work that will lead to a series of calculations for keeping a record of the assets of each player. Thus the player will have the information at every stage to help him in his decision for the next steps

In this lesson it is proposed

(a) to provide the students with information up to a certain stage and ask them to use it for identifying the total value of their assets up to this point

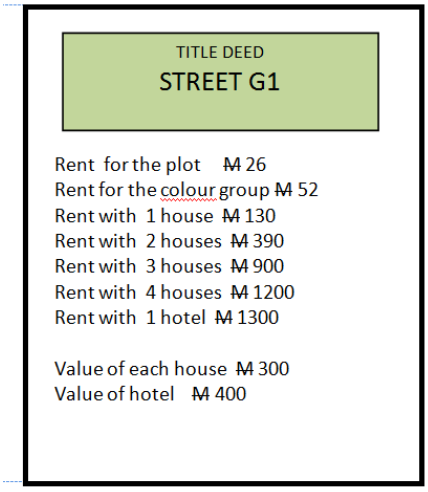
(b) Help them to devise strategies for the next stages in order to increase their assets or to avoid bankruptcy

WORKSHEET 3.3.3 (TEACHER)

Questions/ issues for discussion/ reflection	**Comments/ remarks**
Devise worksheets for the students in order to enable them (a) To calculate the value of their assets and obligations up to each stage of the game (b) To devise plans for either increasing their assets or avoid bankruptcy	Suggest how they organize their possessions by properly arranging their money, titles of deeds etc. Help them to construct tables that will provide information on totals of assets and obligations

WORKSHEET 3.3.3 (LEARNER)

Develop by your own a work sheet for the student using the above mentioned necessary calculations. Here is an example:

Information	**Tasks**
	Given that you have the following banknotes: 2 × ₼ 500, 3 × ₼ 100, 2 × ₼ 50, 3 × ₼ 20, 3 × ₼ 10, 1 × ₼ 5 und 5 × ₼ 1. Calculate: What banknotes do you pay for the rent if a house has not yet been built? What banknotes could you use to buy the street? What does it cost to build the entire 3 streets of a group of 2 houses? What does it cost if I want to build a hotel on a street? What does the rent cost if there is a hotel on the street?

4.1 Petanque (Outside Game)

4.1.1 Lesson 1

Objectives

- The participants learn to construct a line by 2 points.
- The students learn that through 2 points can pass only one line.
- The students learn that through one point can cross infinity of lines.
- The students learn that 2 lines can cross each other in only one point.

Tools, Materials and Organisation

- Petanque is a game that is usually played in an open space but for the purpose of the lesson it will be adapted to be played in a classroom. This way the students will learn about lines and in the meantime learn the fundamentals of Petanque.
- It will require a jack, a circle, as many boules as possible and a tapeline.

Description of the Lesson

First part of the lesson (30 minutes)

- The teacher will place the circle and will show the students how to throw the jack and the boules
- The participants will be divided into groups of 5.
- First of all the teacher will throw the jack.
- The team that throws the boules the closest to the jack wins.
- While one team throws, the others will measure the distances and keep the score.

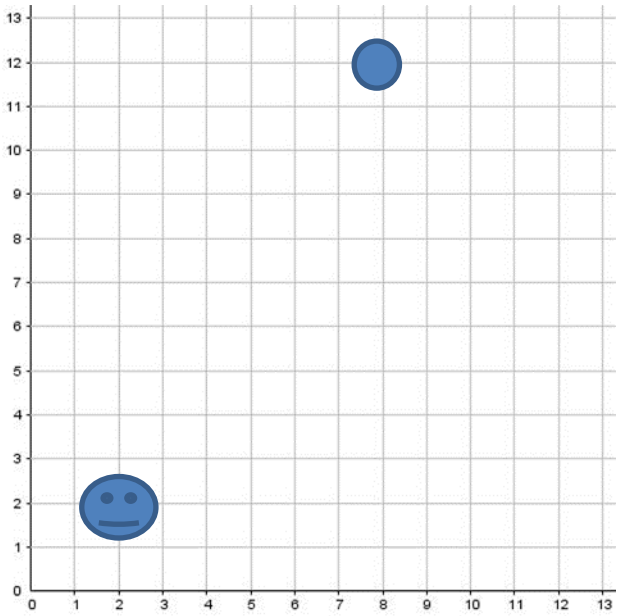
Second part of the lesson (20 minutes)

- Hand out the worksheets: one per person.
- Follow the instructions on the worksheet.
- If there is enough time, one can speak about the differences between indoor and outdoor Petanque.

Useful Hints

- You have to bear in mind that the students have to be able at the least to count and to make additions.

4.1.1 WORKSHEET (LEARNER)



What is the shortest way from the player to the jack?

_____

_____

Take a ruler and a pencil and connect these two points by a straight line.

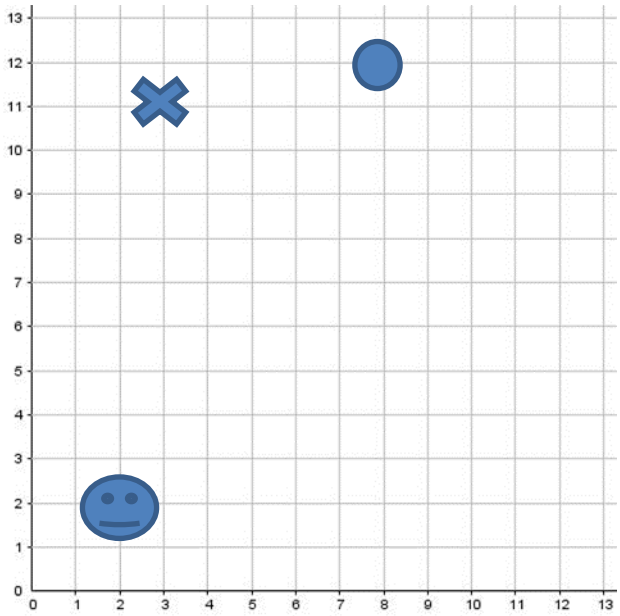
_____

_____

Is that a line or a line segment?

_____

_____



How far can the line that connects the player and the jack go?

_____

_____

How many lines can contain the X point?

_____

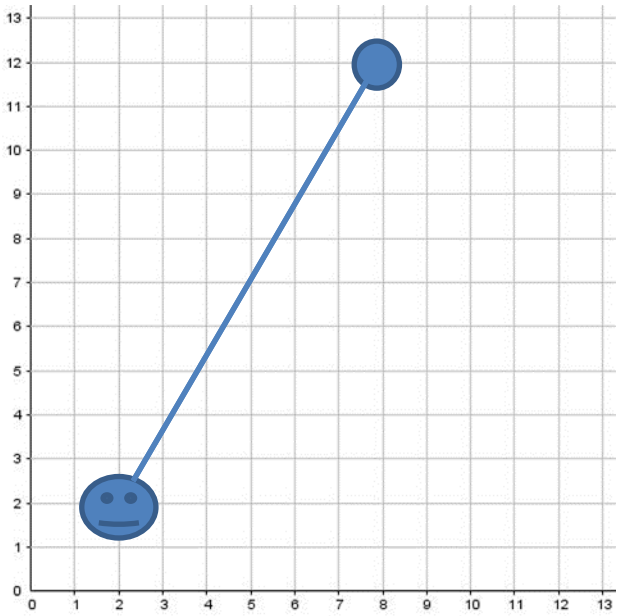
_____

How many lines can contain three non-collinear points such as the player, the jack and X?

_____

_____

4.1.1 WORKSHEET (TEACHER)



What is the shortest way from the player to the jack?

A straight line

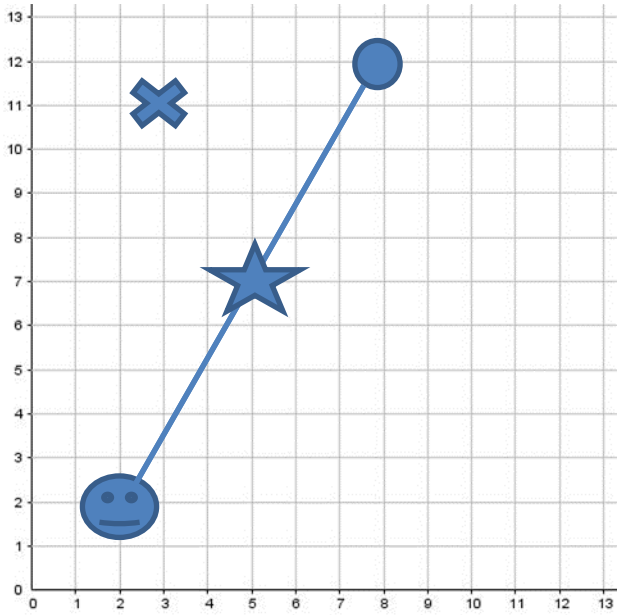
So for every two points A, B there can be no more than one line that contains each of the points A, B.

Take a ruler and a pencil and connect these two points by a straight line.

The line is the shortest way between two points.

Is that a line or a line segment?

It is a line segment, because it is a line with a beginning and an end.



How far can the line that connects the player and the jack go?

Indefinitely = a line contains an infinite number of points.

How many lines can contain the X point?

One point can be contained by an infinite number of lines.

How many lines can contain three non-collinear points such as the player, the jack and X?

At least two distinct lines.

4.1.2 Lesson 2

Objectives

- The participants learn to draw a line by 2 points.
- The students learn to measure the distance between two points.
- The students learn the unit of length: metric system ≠ Imperial system ≠US system.
- The students learn about the metric system = International System of Units (French: Système international d'unités, SI) + non-SI
- The students learn about the meter and its subdivisions.
- The students learn to convert from km to m to dm to cm and to mm.
- The students learn to compare distances.

Tools, Materials and Organisation

- Petanque is a game that is usually played in an open space bur for the purpose of the lesson it will be adapted to be played in a classroom. This way the students will learn about lines and in the meantime learn the fundamentals of Petanque.
- It will require a jack, a circle, as many boules as possible and a tapeline.

Description of the Lesson

First part of the lesson (30 minutes)

- The teacher will place the circle and will show the students how to throw the jack and the boules
- The participants will be divided into groups of 5.
- First of all the teacher will throw the jack.
- The team that throws the boules the closest to the jack wins.
- While one team throws, the others will measure the distances and keep the score.

Second part of the lesson (20 minutes)

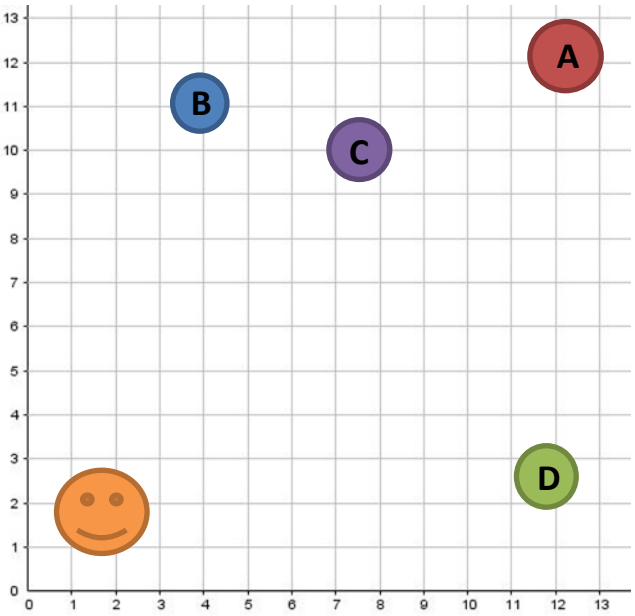
- Hand out the worksheets: one per person.
- Follow the instructions on the worksheet.
- If there is enough time, one can speak about the differences between indoor and outdoor Petanque.

Useful Hints

- You have to bear in mind that the students have to be able at the least to count and to make additions.



4.1. 2 WORKSHEET (LEARNER)



Note: The drawing is not to scale to the distances indicated below!

Which of the players seems to have thrown his boule the closest to the jack (the red boule A)?

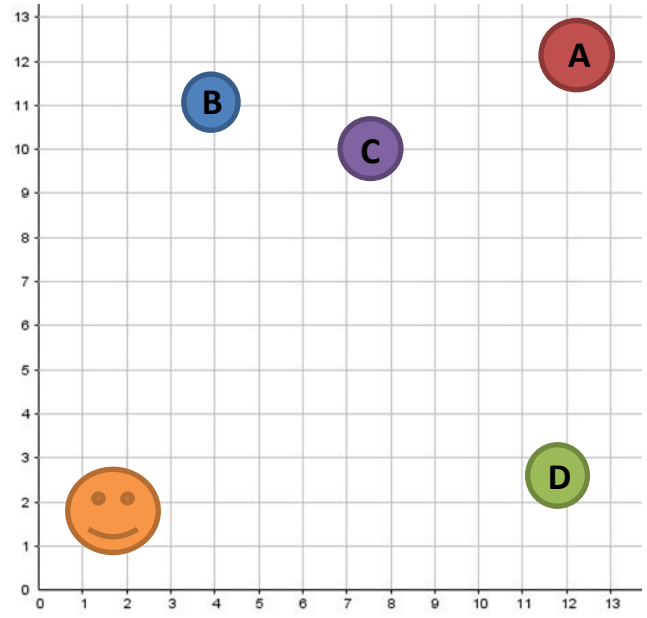
_____

_____

The blue player B has thrown 1.5 m, the purple player C 200 cm and the green player D 10 dm. How many mm is the longest throw?

_____

_____



From the blue ball B to the jack there are 70 cm, from the purple boule C 30 cm and from the green boule D 90 cm. Which one is the furthest from the jack?

_____

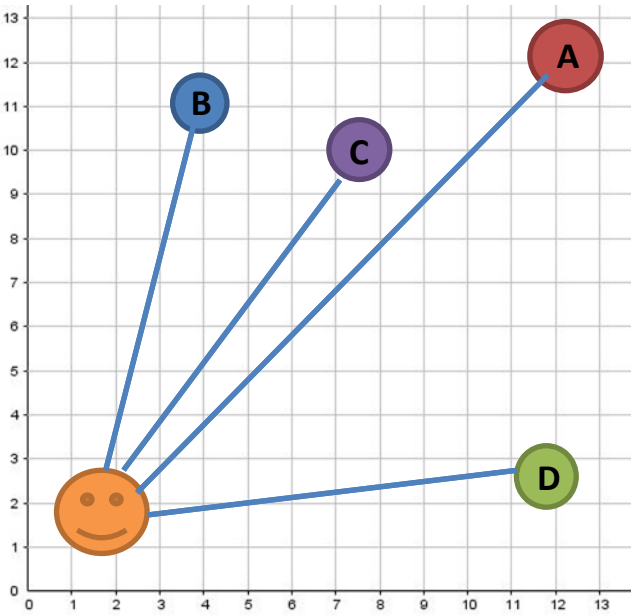
_____

How many m and how many cm are in one km?

_____

_____

4.1. 2 WORKSHEET (TEACHER)



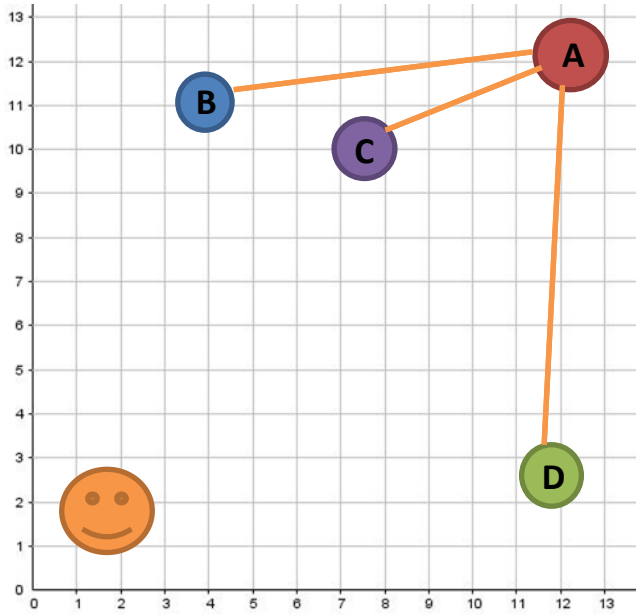
Note: The drawing is not to scale to the distances indicated below!

Which of the players seems to have thrown his boule the closest to the jack (the red boule A)?

The purple boule 3.

The blue player B has thrown 1.5 m, the purple player C 200 cm and the green player D 10 dm. How many mm is the longest throw?

1.5 m = 1500 mm, 200 cm = 2000 mm, 10 dm = 1000 mm so the purple B is the longest.



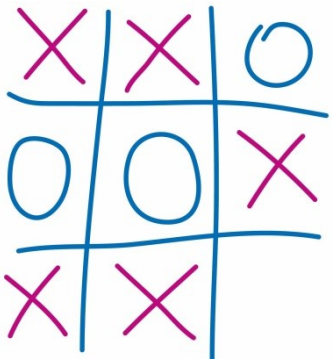
From the blue ball B to the jack there are 70 cm, from the purple boule C are 30 cm and from the green boule D are 90 cm. Which one is the furthest from the jack?

The green boule D with 90 cm.

How many m and how many cm are in a km?

1 km = 1000 m = 100000 cm

4.2 TICKTACKTOE (PAPER-PENCIL GAME)



4.2.1 LESSON 1

OBJECTIVES

- Participants will be taught to count reliably up to 9 items.
- They will be able to read and identify any single digit number even if it is not in a consecutive order.
- Understand a coordinate system and find positions up to 10
- They will know how to count on and back from any small number. They will learn the idea of a number line.

TOOLS, MATERIALS AND ORGANISATION

- A big ticktacktoe board with detachable boxes for the teacher.
- One extra box for the teacher.
- Smaller ticktacktoe boards with detachable boxes for groups of 4 students.
- Prepare copies of the worksheet for each student. The lesson takes 45 minutes.

DESCRIPTION OF THE LESSON

First part of the lesson

- Explain the ticktacktoe game
- Build up groups of 4 persons.
- Each group is sitting at a table.
- The role of each person in the group: two students are playing while the two others are watching. After one round they will shift roles. They will use pen and paper, not the detachable boards.
- The participants play several times the game. They start developing strategies. They will experience a no winner situation. This makes them feel in control and is fun.
- The participants learn to count and to read up to 9 by putting boxes on the board.

Second part of the lesson

- Hand out the detachable boxes boards
- Ask them to write the numbers from 1 to 9 on each box
- Put the teachers' boxes in consecutive order from 1 to 9 on the black board
- Ask the students to play ticktacktoe using the numbered boxes (in the place of X and O)

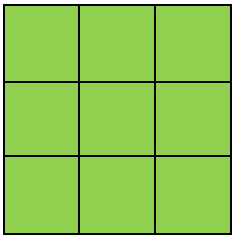
Third part of the lesson

- Hand out the worksheets: one to each person. Follow the instructions on the worksheet.
- The participants learn that numbers have an order and build up a number-line.
- Add "0" to the order and the number line.

USEFUL HINTS

- If the participants cannot read, the teacher has to guide them.
- If participants have difficulties to count, you have to train – you need more time or you can split the group.
- If participants have difficulties to write numbers, you have to divide the lesson into two lessons: First lesson: play and count, second lesson: play and write numbers.
- Following lesson: Using the number boxes learn to count up to 100 and more.

4.2.1 WORKSHEET (LEARNER)



How many boxes do you see on the picture on the left?

_____

_____

X	O	O
X	X	
X		O

How many X and how many 0 do you see in the picture?

_____

_____

1		3
	5	6
7		9

What are the missing numbers in chronological order? Fill them in.

_____

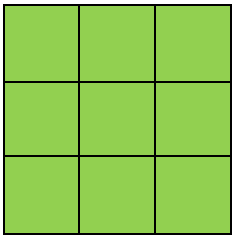
_____

Put a number in each box in the right order!

Draw a line number starting with 0.

_____

4.2.1 WORKSHEET (TEACHER)



How many boxes do you see on the picture?

9

X	O	O
X	X	
X		O

How many X and how many 0 do you see in the picture?

4 X and 3 O

1	2	3
4	5	6
7	8	9

What are the missing numbers in chronological order? Fill them in.

2, 4, 8

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*

Put a number in each box in the right order!

Draw a line number starting with 0.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

4.2.2 LESSON 2

OBJECTIVES

- Participants will be taught to recognize and name two-dimensional shapes: the square and the rectangle. The Ticktacktoe game was chosen, because the game is played in a series of squares, which can be transformed into rectangles.
- They will be able to identify the geometrical figures and to differentiate the square from the rectangle.
- They will also be able to describe and understand length and width of shapes.
- They will learn others elements of a rectangle and a square: angle and diagonal.

TOOLS, MATERIALS AND ORGANISATION

- A big ticktacktoe board with detachable boxes for the teacher.
- One extra box for the teacher.
- Smaller ticktacktoe boards with detachable boxes for groups of 4 students.
- Prepare copies of the worksheet for each student.
- The lesson takes 45 minutes.

DESCRIPTION OF THE LESSON

First part of the lesson

- Explain the ticktacktoe game
- Build up groups of 4 persons.
- Each group is sitting at a table.
- The role of each person in the group: two students are playing while the two others are watching. After one round they will shift roles. They will use pen and paper, not the detachable boards.
- The participants play several times the game. They start developing strategies. They will experience a no winner situation. This makes them feel in control and is fun.

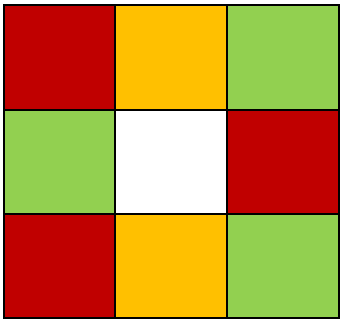
Second part of the lesson

- Hand out the worksheets: one for each person.
- Follow the instructions on the worksheet.
- Exemplify the square: the figure used to begin the game and the boxes inside
- Creating a rectangle: 2 squares become a rectangle.
- Explain the length and the width of both rectangle and square by using two of the win situations.
- Insist on the difference between the two geometrical forms
- Explain the diagonal by using one of the win situations.
- Explain and exemplify the angle.

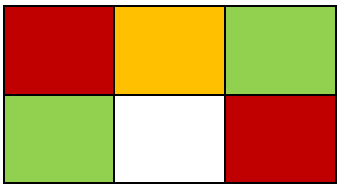
USEFUL HINTS

- If the participants cannot read, the teacher has to guide them.
- The participants have to have an idea of what geometry is. It is not really an introduction to geometry lesson.
- Following lesson: By replacing the 0 with a circle by similarity they will be able to also recognise a circle.

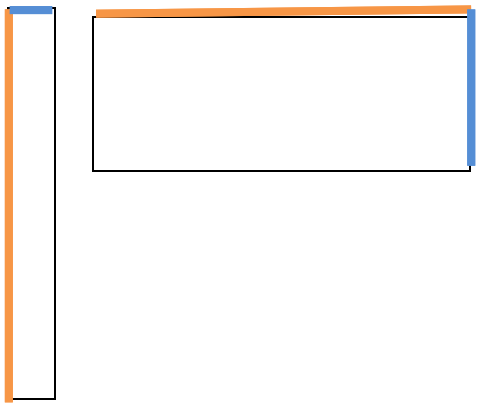
4.2.2 Worksheet (Learner)



1. How many squares do you see in the picture left?

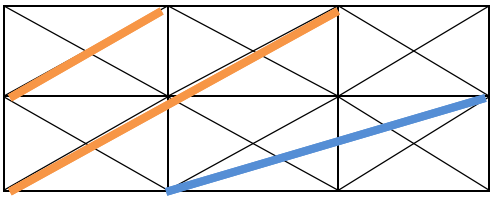


2. How many squares and how many rectangles do you see in the picture left?



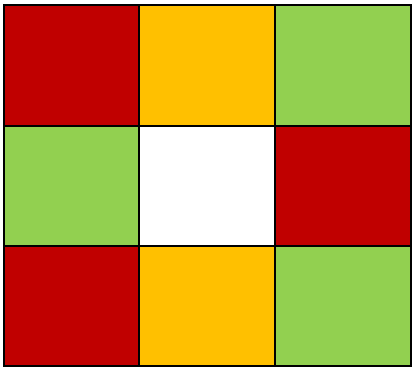
3. Could you please identify in the pictures left the length and the width? Please mark the length in red and the width in blue and measure it in cm!

A Petanque field in geometric rectangle form



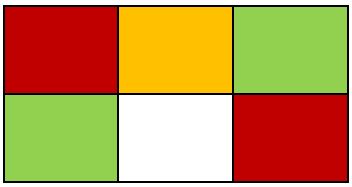
4. Identify all the diagonals in the pictures left. How many are there?

4.2.2 WORKSHEET (TEACHER)



1. How many squares do you see in the picture left?

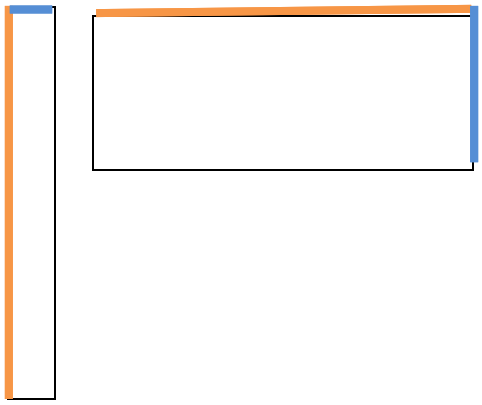
There are 14 squares, 9 small ones, 4 bigger ones and 1 biggest one.



2. How many squares and how many rectangles do you see in the picture left?

Squares: 8

Rectangles: 10



3. Could you please identify in the pictures left the length and the width? Please mark the length in red and the width in blue and measure it in cm!

red=length:

left rectangle 5.2 cm,

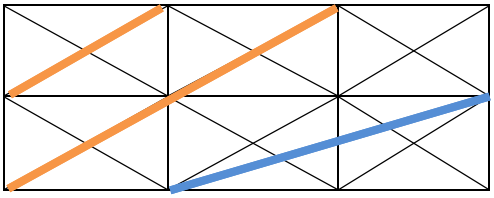
right rectangle 5 cm

blue=width:

left rectangle 0,7 cm,

right rectangle 2 cm

A Petanque field in geometric rectangle form



4. Identify all the diagonals in the pictures below. How many are there?

There are 12 diagonals painted in 6 rectangles. If you identify bigger rectangles, you see more diagonals.

4.3 ROCK-PAPER-SCISSORS (EVERYWHERE GAME)

OBJECTIVES AND HINTS

- The simplest definition of probability: "Probability is a branch of mathematics that deals with calculating the likelihood of a given event's occurrence, which is expressed as a number between 1 (certainty) and 0 (impossibility). 0.5 is the odds possibility.
- Try to control a probability. If it is possible and on what conditions.
- Identify a strategy and design a strategy.
- Games of chance vs. gambling: identification and awareness.
- Uses of probabilities in day-to-day life: statistics, computer programming, astrophysics, music, weather forecast, medicine.

- Learn the simplest way of calculating a probability. *In its simplest form, probability **p** can be expressed mathematically as the number **m** of occurrences of a targeted event divided by the number **n** of possible occurrences*:
  p = m/n
- **Example 1:** Throwing a coin (head/number), what is the probability to get a "head"? (m=1, n=2):
  p = 1/2 = ½ = 0.5 (it means, it is odds possibility to win)
- **Example 2:** Game rock, paper, scissors, what is the probability to win? (m=1, n=3):
  p = 1/3 = 0.33
  (it means, the possibility to win is one-third)
- **Example 3:** Roll the dice, what is the probability to roll a 6? (m=1, n=6): p = 1/6 = 0.166 (it means, the possibility to win is one-sixth)

TOOLS, MATERIALS AND ORGANISATION

There is no need for great preparation but the students must have a solid knowledge of basic arithmetic.

Tools:

- A coin to exemplify the probability in the simplest of ways (two possibilities). Rock-paper-scissors is more complex because there are three possibilities. We think that is the next step in understanding the concept and its complexity.
- A blackboard or a flipchart to keep the scores and make the calculations.

DESCRIPTION OF THE LESSON

Due to the complexity of the concept one lesson is not enough. At least two lessons are needed.

First lesson

- The teacher will chose two volunteers and ask them to play "heads" or "tails" with the coin.
- He will keep the scores.
- After several attempts he will ask the others students to predict the result.
- The teacher explains the definition of probability and the simplest calculation method.
- In using the calculation formula and the scores from "heads" or "tails" the students will calculate the probability.
- In the end the teacher will present different games of chance.

Second lesson

- The teacher explains the rules for the rock-paper-scissors game.
- The students will play it in pairs and keep the score. They will thus understand that rock-paper-scissors is more complicated than the coin game.
- The teacher will explain the complexity of the probability games and the fact that in the rock-paper-scissors game there are three possibilities vs. two in the coin game.
- The students will be asked to calculate the probability and try to find a method to increase the probability of winning.
- The teacher will then explain the concept of strategy and its limits in connection to gambling.
- In the final stage, there will be a discussion about the usefulness of probabilities in real life.
- The three tables help you with the game.

USEFUL HINTS

- In presenting and discussing the strategy, the teacher can also use the example of the Ticktacktoe game where it is easier to form a strategy.
- In speaking about chance games vs. gambling and limits of the strategy, the teacher can also use the example of the Black Jack game.

5.1 Mensch Ärgere Dich Nicht (Engl. Ludo)

Preliminary Remark

The board game is already mentioned as a Bulgarian variant “Man don’t be angry” in section 2.2. with a description of the game and worksheets for teachers and participants. The description and worksheets might be adapted and used for 5.1 as well. The rules are comparable.

Objectives

- Count reliably up to 6 items
- Read and write numbers up to 40
- Order, compare and add numbers up to 6,
- Subtract single-digit numbers from numbers up to 6
- Coordinate different figures in their positions (holistic overview)
- Compare, count and organize own and other pieces
- Experiencing and training in complexity (in groups of 2-6 players)
- How to deal with emotions in small groups

Tools, Materials and Organisation

- A group of 2 to 6 persons is sitting around the board on a table
- Each person has his four pieces in one of the colours: Black, yellow, red or green on the game board with four sections. Or: Black, yellow, red, green, blue or violet on the game board with six sections
- Training in complexity and experiencing the differences: Two persons are playing with eight pieces, four persons are playing with 16 pieces, six persons are playing with 24 pieces.
- Participants make notes of the results and annotations about the feelings when playing with two, four or six players.

Description of the Lesson

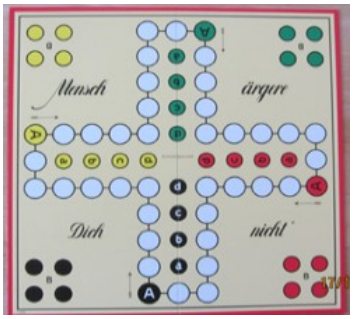
First part of the lesson (10 minutes)

- Learning the rules: Make a group of 2 to 4 participants.
- Play the game several times and note the winner.

Second part of the lesson (35 minutes)

- Playing the game with two persons
- Playing the game with four persons
- Playing the game with six persons
- Writing notes by the participants
- Discussion about the “training in complexity” in the whole group

Useful Hints



Another variant for a “Training in complexity”: An even more complex “reverse” version can be played by racing the pieces back to the starting field (by not more than four players!). Question: Which kind of strategies are needed for this “reverse variant” (E.g. this version was played in a family by four people just to have more fun. Additional rule: A Six is needed before going back from field A into the starting field B). The board game could be played with even or odd numbers of participants.

5.2 Seven Steps (German Dance)

Objectives

- Participants should be taught to count reliably up to 8 items. The dance Seven Steps was chosen as a moving game. Seven is the biggest number of danced steps and eight the biggest number of music bars (music clock).
- They should understand the difference of the dance rhythm with seven performed steps and count the eight times (quaver time/Achteltakt).
- The dance "Seven steps" combines counting and movement. It was chosen, because of its "loosen up" and its emotional quality.
- They should identify two-dimensional shapes of steps on the floor or painted on a paper.
- Participants should learn to count up to 7 by dancing in the first phase, up to 3 in the second phase, up to 4 in the third phase.

Tools, Materials and Organization

- Free space for dancing
- Danced by two persons in a row or in a circle
- Prepare copies from the worksheets
- The lesson has two parts and takes 90 minutes

Description of the Lesson (step-by-step approach)

First part of the lesson

- Experiencing the melody and the steps: Sing the song several times in the group with the text, which is at the meantime explaining the steps. Alternative: an instrument plays the melody, e.g. guitar or flute; or download the song from the Internet.
- Rhythmic walking with short steps and long steps: Phase a: seven steps; Phase b: four steps. Counting up to 7, 3, 4 in the different phases of the dance.
- Arrange the dancing group with couples in a row or in a circle.
- Performing altogether the three phases of the dance in an easygoing way – just to have fun by moving, counting and singing. Making mistakes is just a part of having fun.
- Analysing the dance in two groups: One part of the learners is dancing, the other part is watching.
- Discussing and experiencing the steps in the whole group (e.g. 7th step as equivalent for tact 7+8).
- Perform the dance several times in the group. Task for the watching group: Describe the dance with your own words (partner work).
- Worksheet #1. Form to be filled in by the participants (single or partner work).

Second part of the lesson

- Explication of different ways of showing quantities (in a line - horizontal or vertical, in a circle, a cross etc.).
- Different ways of designing a melody (form with five lines, by forming a wave with the hands moving up and down, designing a graph line with two variables: height and time
- Worksheet #2 is handed out to each participant
- Participants make lines around different quantities. Participants fill in the empty graph line
- Discussing open questions and performing the dance at the end of the lesson

Useful Hints

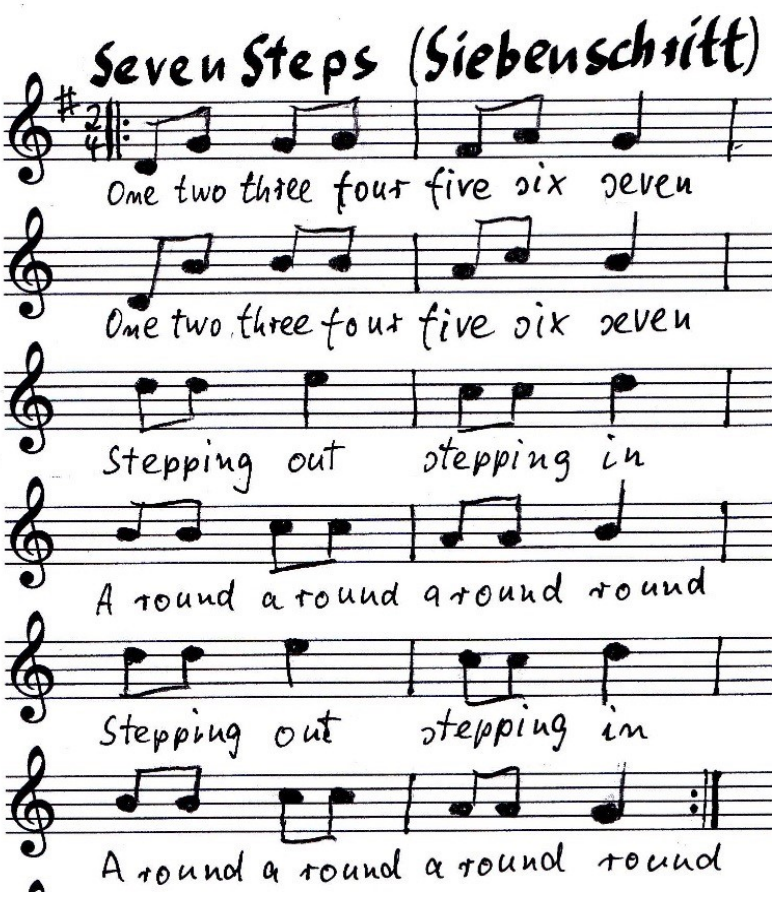
- The dance can be an ice-breaking in other lessons.
- The graph line is related to music and the writing of notes. So it may be combined with a music lesson.
- It is difficult to combine different movements and rhythmic systems steps on the ground clapping of hands singing a melody or counting aloud. It may be regarded as a quite ambitious pedagogical approach
- More information in the Internet, e.g. TaKeTiNa (https://en.wikipedia.org/wiki/Taketina)

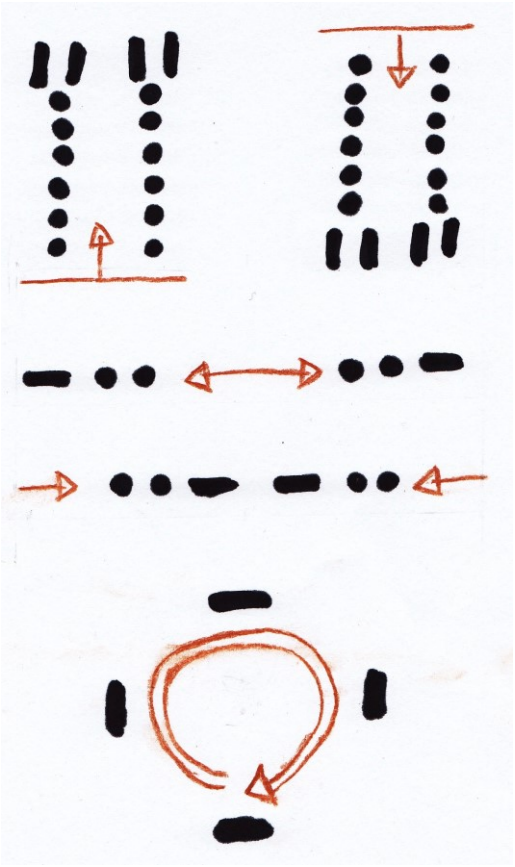
WORKSHEET 5.2 PAGE 1 (LEARNER)

SING THE SONG & EXPERIENCE THE RHYTHM

After having danced you may discuss with your neighbours in a small group the structure of the dance and the three phases

- The notes of the melody (shown on the left)
- The steps of the rhythm (shown on the right)
- Which rows of the melody and which parts of the steps are related to the three phases of the dance? Mark it with a, b, c (with a circle and/or different colours).





Describe and fill in the numbers:

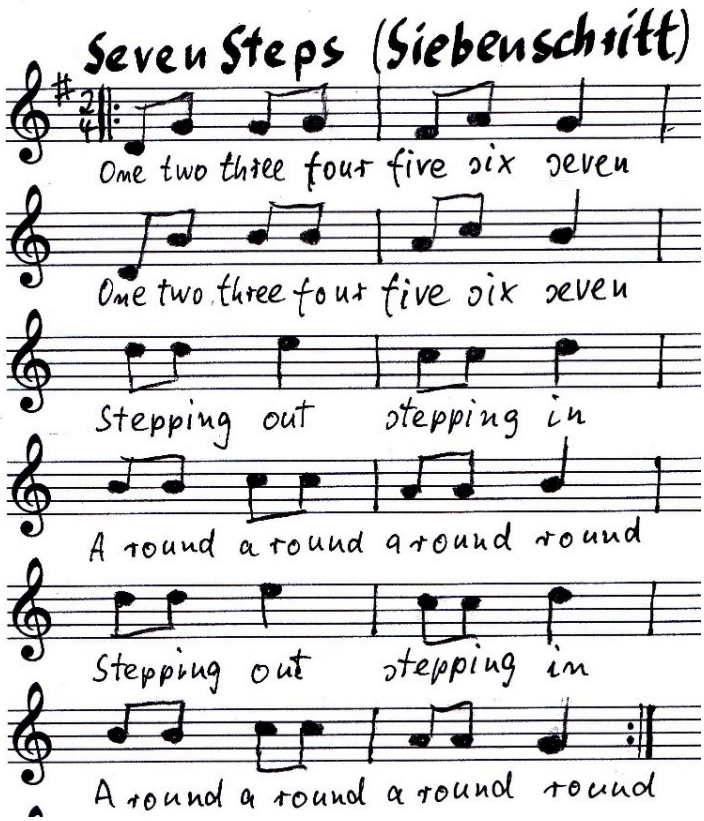
- How many rhythmic steps in each of the three phases?
- How many clapping hands (following the tact) in each of the phases? Fill in the form and discuss it with your neighbours.

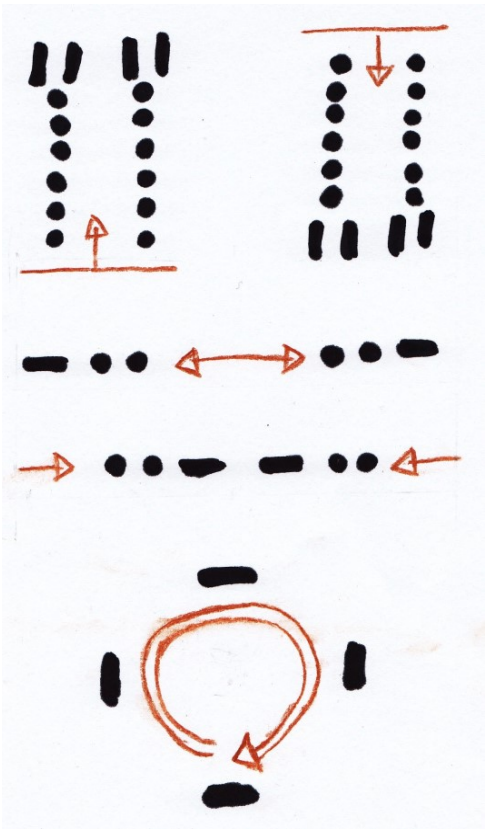
Phase	Number of steps: Short Long	Number of tacts: clapping hands	Remarks
a			Twice: Foreward and backward
b			Twice: Outside and inside
c			Each couple is dancing in a circle

WORKSHEET 5.2 PAGE 1 (TEACHER)

NOTES & STEPS, SING THE SONG & EXPERIENCE THE RHYTHM

Which rows of the melody and which parts of the steps are related to the three phases of the dance?
Part (a) of the steps graphic — rows 1 and 2 of the notes
Part (b) of the steps graphic — rows 3 and 5 of the notes
Part (c) of the steps graphic — rows 4 and 6 of the notes





Further explanation to the form: Phase (a): 7 steps, phase (b): 3 steps, phase (c): 4 steps

Phase	Number of steps: Short	Long	Number of tacts: clapping hands	Remarks
a	6	1	8	Twice: Foreward and backward
b	2	1	8	Twice: Outside and inside
c	0	4	8	Each couple is dancing in a circle

Additional:
LONG — SHORT in other dances
Students may describe the rhythm of other dances of their choice e.g. Sirtos or Kalamatianos.
Short — short — long (This type of rhythm is called in poetry „Anapest“)
Long — short — short (This type of rhythm is called in poetry „Dactylous“)
Seven Steps in phase (b) with the rhythm “Short — short — long” is an Anapest and Sirtos or Kalamatianos with the rhythm “Long — short — short” is a Dactylous.
Another example of counting: Foxtrott has the rhythm “Long — long — short —short”
According to theories of poetry and music, the anapest-rhythm is energizing and the dactylous-rhythm is harmonizing.

WORKSHEET 5.2 PAGE 2 (LEARNER)

Task 1

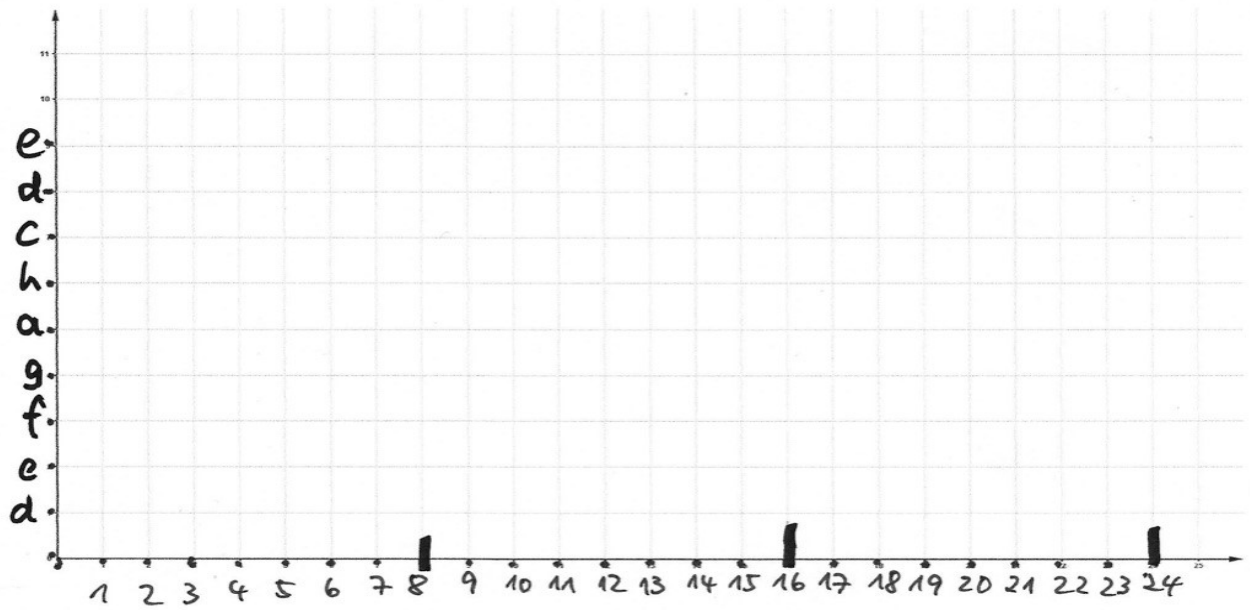
Mark the numbers of dancing steps 7 - 3 - 4 by making lines for different quantities

Task 2

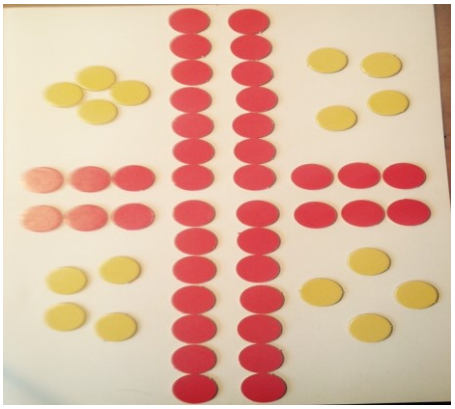
Construct a line graph with two variables

HEIGHT (tones/notes) & TIME (tact unites)

- Move your hands up and down following the melody
- Combine the movement of the hands up and down with making the steps (with the feet on the ground the floor in an easy going manner).
- Fill in the empty graph line with „waves“ of the melody (single or partner work)
- Discuss the results with your neighbours: E.g. common and different findings, difficulties, questions



WORKSHEET 5.2 PAGE 2 (TEACHER)



Task 1

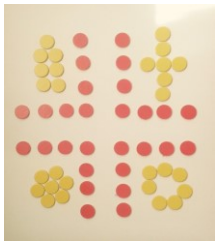
For developing a sense of structure the students can identify and mark the numbers of dancing steps 7 - 3 - 4 by making lines for different quantities

- Seven pieces - arranged vertically (four times: 4x7=28)
- Three pieces - arranged horizontally (four times: 4x3=12)
- Four pieces - arranged in the four corners (four times: 4x4=16)
- Altogether 56 pieces (28+12+16)

Additional:

Playing with different quantities related to the dancing steps and with different background. Students may create their own arrangements.

Here two examples: different quantities and a polychrome background.



Task 2

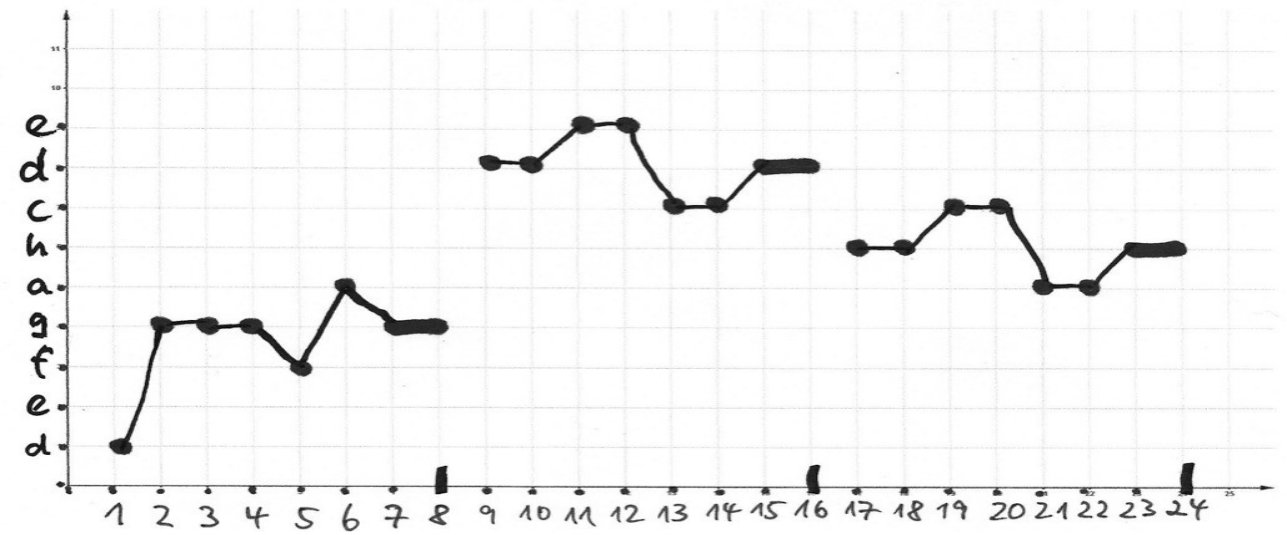
How to show a melody? Notes are a common form used in music.

There are other possibilities:

1. Hands moving up and down following the melody.
2. Combining the movement of the hands up and down and making the steps with the feet on the ground (just for „loosen up“ as W. Meyerhöfer (see Math-GAMES Compendium Prologue) remarked, experiencing 2-D and 3-D on the floor and in the space in an easy going manner).
3. Constructing a line graph. (Time and height as variables of the diagram.)

Constructing a line graph, Variables: HEIGHT (tones/notes) & TIME (tact unites)

Height

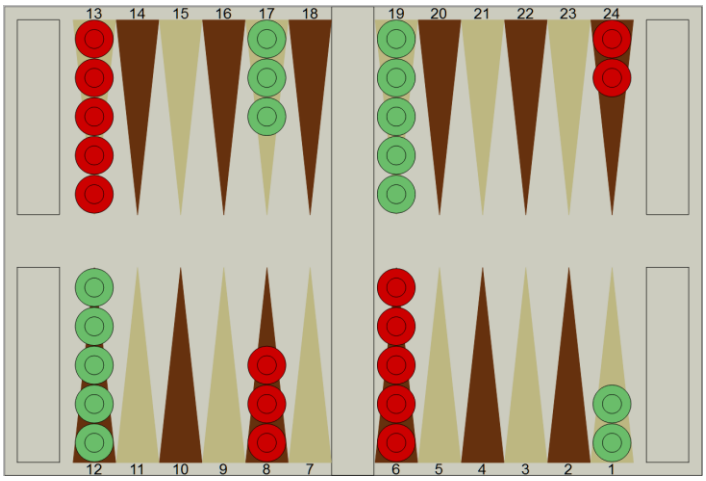


Sheet with melody: 1st line 3rd line 4th line Time

Students may form the wave of the melody first with hands in the air. Then they are informed about the notes: lowest note of the melody (d) and highest note of the melody (e), and then fill in the empty form - in single work or partner work.

6.1 BACKGAMMON (BOARD GAME)

OBJECTIVES



- Students identify the range of possible outcome when using one or two dices
- Students should learn to count and add single-digit numbers with totals to 36
- Students learn to multiply using single-digit whole numbers

TOOLS, MATERIALS AND ORGANIZATION

- Take a backgammon board for each two players
- Prepare copies of the Worksheets for each student
- The lesson takes 45 to 60 Minutes or more

DESCRIPTION OF THE LESSON

First part of the lesson

- Preparation – introduction
- Short connection with the previous lesson and description of the game.

Second part of the lesson

- Presentation about the coordinates, the dice and the movements
- Work on the worksheet
- Students join groups (groups of 2 or 3). Each group has a backgammon board. They can use the backgammon to help them with the worksheet. The students fill the exercises in the worksheet.

Third part of the lesson

- Teacher will check the answers and discuss the results with the students.
- Summarize

USEFUL HINTS

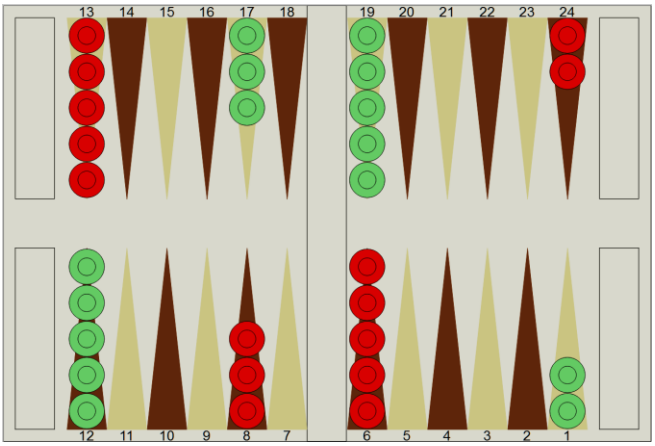
Useful links:

- You can download a backgammon game with several variants from http://ai.uom.gr/nikpapa/Palamedes/

Follow up hints:

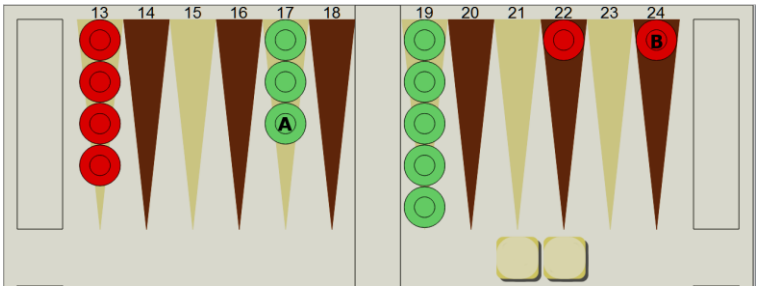
- Ask students to play at least two variants of a backgammon. Then, discuss the differences and the level of difficulty between the backgammon variants.

WORKSHEET 6.1 (LEARNER)

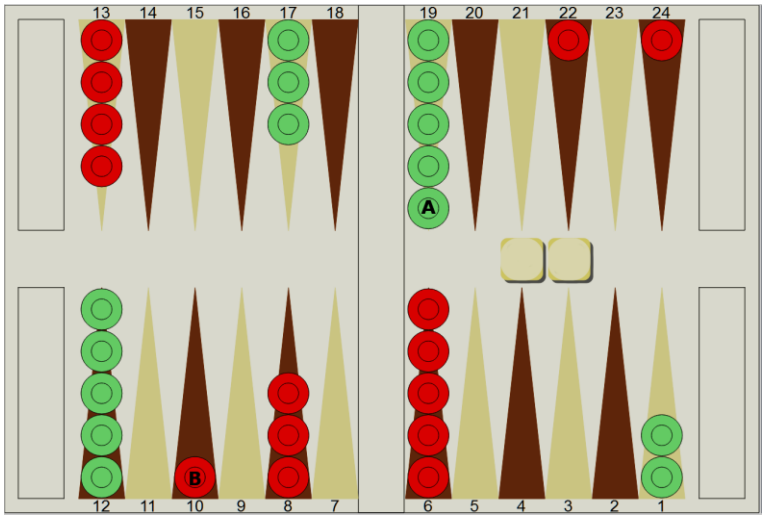


EXERCISE 1

Task A



Task B



INTRODUCTION

Count the checkers for each colour. Are they the same number?

_____

_____

EXERCISE 1

Which roll of the dices do you need so that checker A will hit checker B?

Task A.

_____

_____

Task B.

_____

_____

In task B, is it an option the roll of the two dices: 3 and 3?

Answer: _____

EXERCISE 2

Write the total number of movements for each roll:

A. 5 and 6 _____

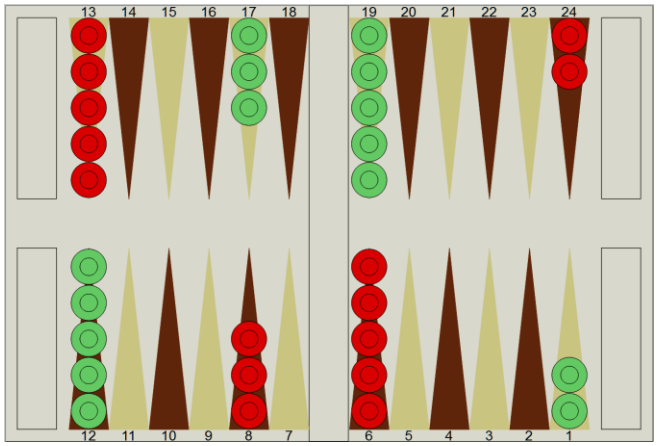
B. 3 and 1 _____

C. 4 and 4 _____

D. 2 and 5 _____

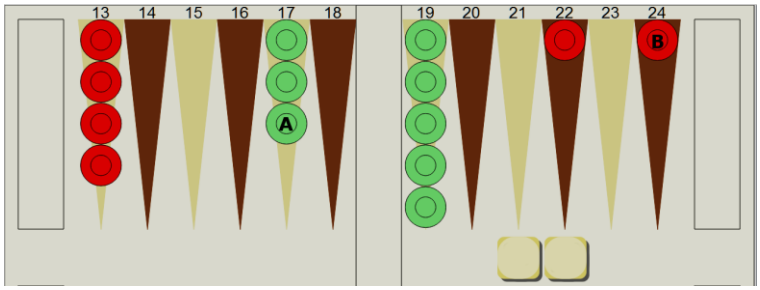
E. 6 and 6 _____

WORKSHEET 6.1 (TEACHER)

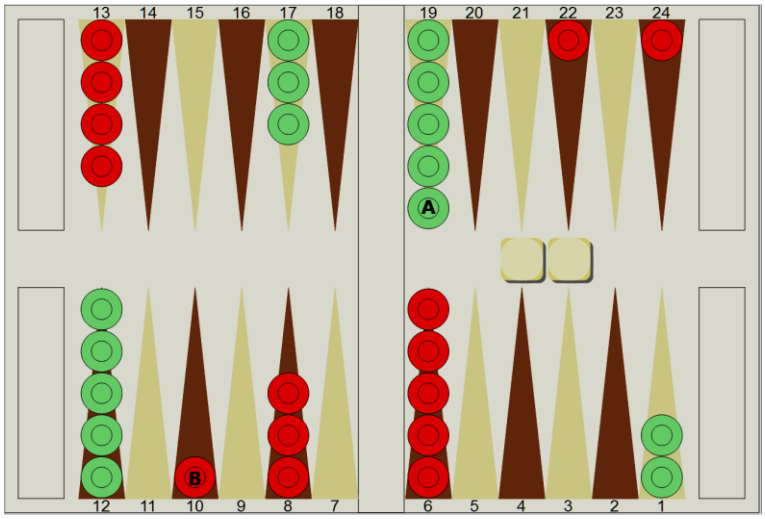


EXERCISE 1

Task A



Task B



INTRODUCTION

Count the checkers for each colour. Are they the same number?

Red: 15, Green: 15

Yes, they are the same.

EXERCISE 1

Which roll of the two dices do you need so that checker A will hit checker B?

Task A.

I roll with two dice 6 and 1, 4 and 3, 5 and 2, 1 and 6, 3 and 4, 2 and 5

Task B.

6 and 3, 5 and 4, 4 and 5, 3 and 6

In task B, is it an option the roll 3 and 3?

Answer:

No, it hits on 13-point

EXERCISE 2

Write the total number of movements for each roll:

A. 5 and 6: *5 + 6 = 11*

B. 3 and 1: *3 + 1 = 4*

C. 4 and 4:

4+4+4+4=16 or 4x4=16

D. 2 and 5: *2 + 5 = 7*

E. 6 and 6: *6 x 6 = 36*

6.2 CHESS (BOARD GAME)

OBJECTIVES

- Students should learn the use of coordinates and find positions
- Students understand the value of game-objects
- Students learn to multiply by 3
- Students learn to solve easy equations with symbols

TOOLS, MATERIALS AND ORGANIZATION

- Take a Chess board for each 3 players
- Prepare copies of the Worksheets for each student
- The lesson takes 45 to 60 Minutes or more

DESCRIPTION OF THE LESSON

First part of the lesson

- Preparation – introduction
- Short connection with the previous lesson and description of the game.

Second part of the lesson

- Presentation about the coordinates
- Work on the worksheet
- Students join up groups (groups of 2 or 3). Each group has a chess board. They can use the chessboard and the chess pieces to help them with the worksheet. The students fill the exercises in the worksheet.

Third part of the lesson

- Teacher will check the answers and discuss the results with the students.
- Summarize

USEFUL HINTS

Useful links:

- More exercises in Greek language in http://aesop.iep.edu.gr/node/13670

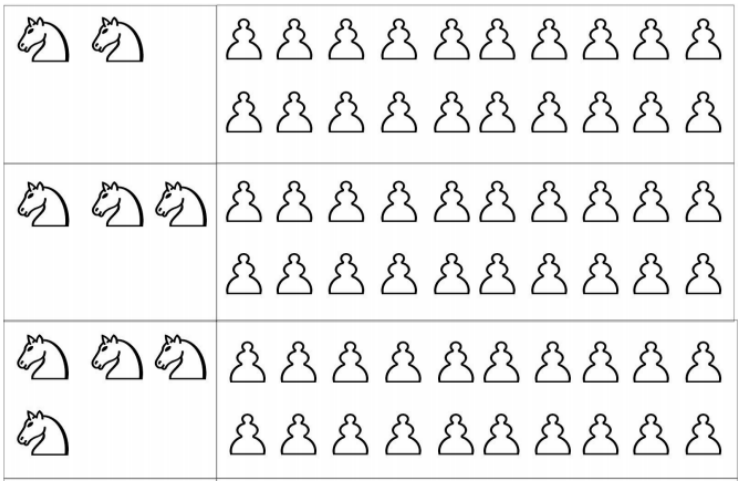
Follow up hints:

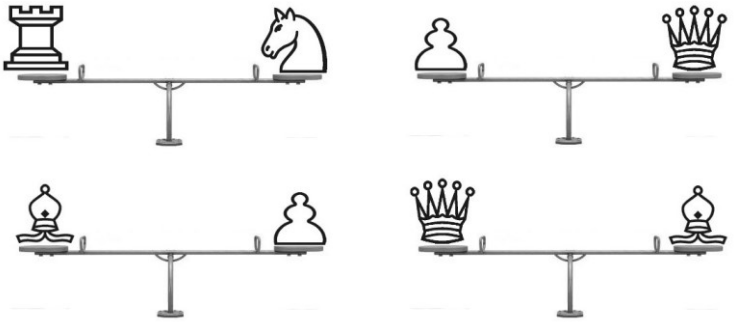
- Ask students to play a chess game and note the coordinates for every move they make.

„Matt in drei Zügen" / „Checkmate in three moves"

German and Jürgen are playing check near Isar river in Munich within a ZDF TV-production.
This event was a nice opportunity to present the Math Games Project and the Math Games Compendium.
30th of August 2016, Titel of the TV production: Murder in Munich („München Mord" Folge: Schuld und Sühne -Arbeitstitel).

WORKSHEET 6.2 (LEARNER)





INFORMATION

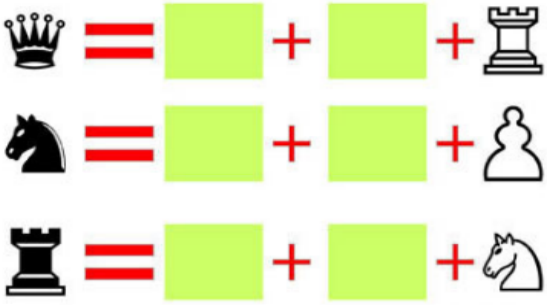
Pawn: value 1
Knight (Horse): value 3
Bishop: value 3
Rook (Castle): value 5
Queen (Lady): value 9
King: value ∞ (infinite)

EXERCISE 1

Knowing, that the Knight is worth as much as three Pawns, draw as much Pawns as needed to have equal power with the Knight in the left column

EXERCISE 2

Circle the piece with the higher power in each of the four pictures!



EXERCISE 3

Complete the following equations! Use the results from the exercises 1 and 2!

EXERCISE 4

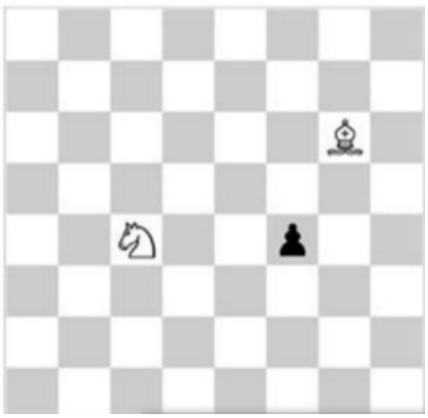
Give each column and each row a number (a coordinate)!

Write the coordinates of the three pieces.

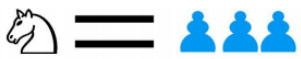
Horse: _____

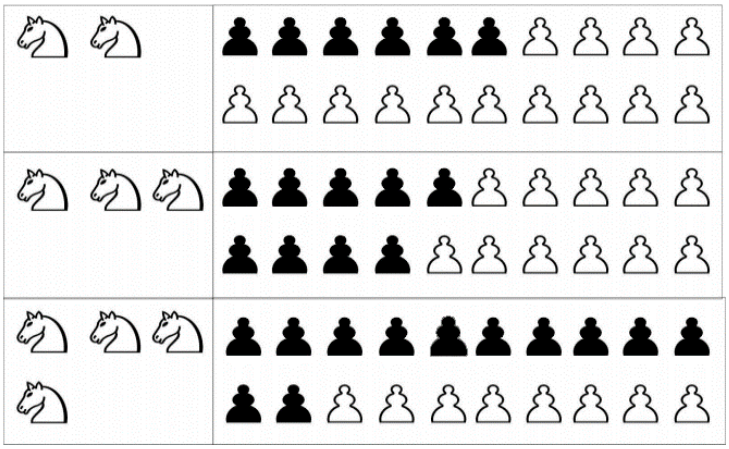
Pawn: _____

Bishop: _____



WORKSHEET 6.2 (TEACHER)





INFORMATION

Pawn: value 1
Knight (Horse): value 3
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Rook (Castle): value 5
Queen (Lady): value 9
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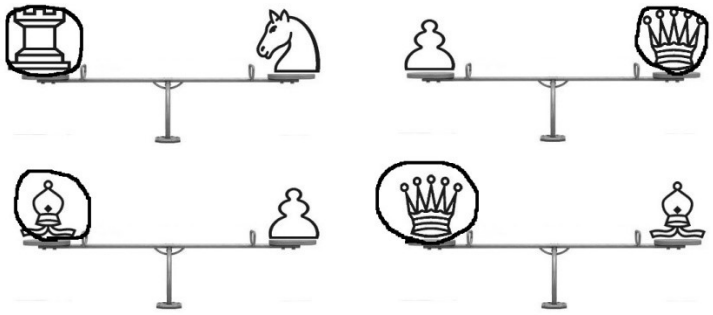
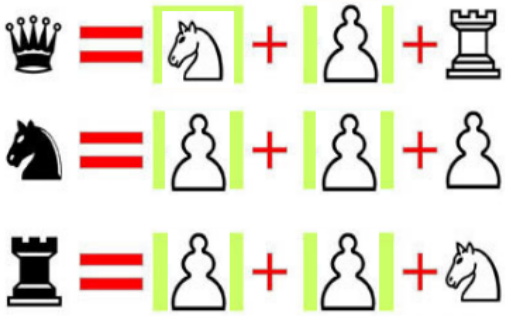


EXERCISE 1

Knowing, that the Knight is worth as much as three Pawns, draw as much Pawns as needed to have equal power with the Knight in the left column

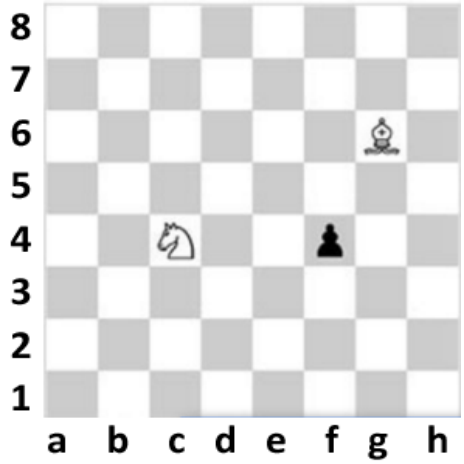
EXERCISE 2

Circle the piece with the higher power in each of the four pictures!



EXERCISE 3

Complete the following equations! Use the results from the exercises 1 and 2!



EXERCISE 4

Give each column and each row a number (a coordinate)!

Write the coordinates of the three pieces.

Horse: *c4*

Pawn: *f4*

Bishop: *g6*

6.3 Calculator Hopscotch (Outside Game)

Objectives

- Students should learn to count and add two-digit whole numbers
- Students should learn to subtract single-digit numbers
- Students learn to solve easy equations with symbols

Tools, materials and organization

- Chalk or a stone to draw the game on the ground
- Prepare copies of the Worksheets for each student
- The lesson takes 45 Minutes or more

Description of the lesson

First part of the lesson

- Preparation – introduction
- Short connection with the previous lesson and description of the game.

Second part of the lesson

- Play the game
- Then, work on the worksheet
- The students fill in the exercises on the worksheet.

Third part of the lesson

- Teacher will check the answers and discuss the results with the students.
- Summarize

Useful hints

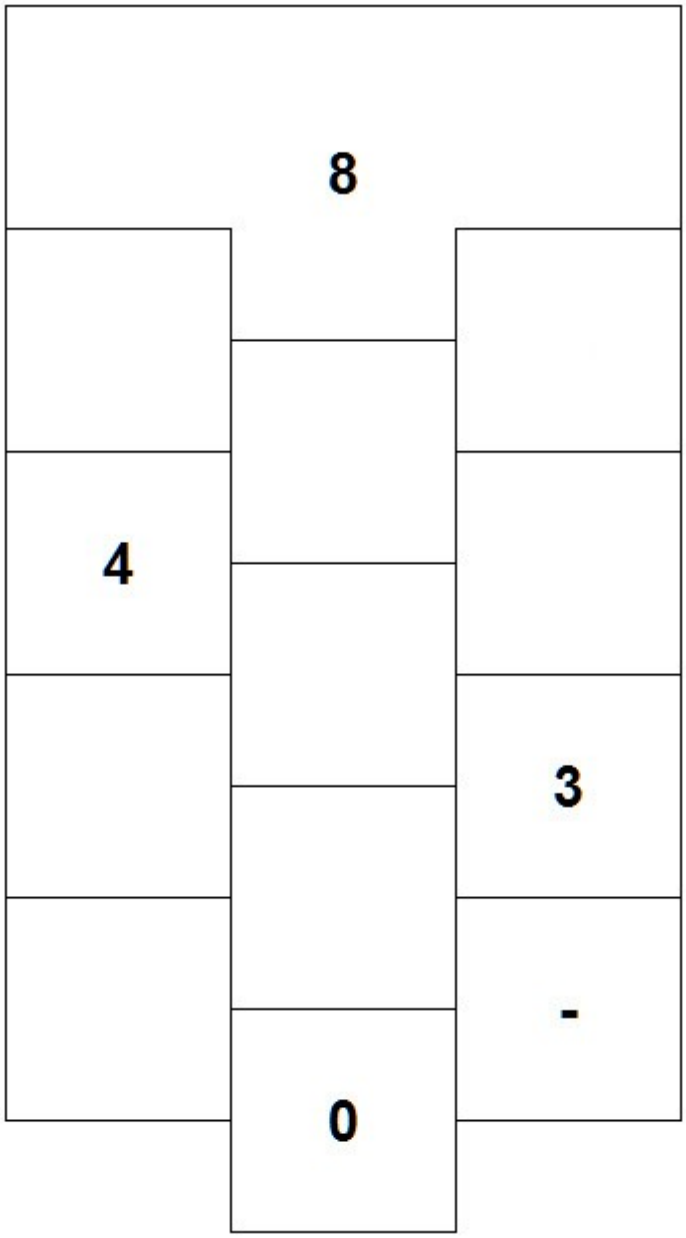
Useful links:

- Hopscotch Game instruction on YouTube Math-GAMES-channel: https://www.youtube.com/watch?v=F81h01Asr7U

Follow up hints:

- Ask students to play the hopscotch based on the examples of the worksheet.

WORKSHEET 6.3 (LEARNER)



INFORMATION

Use the calculator hopscotch as described in the Compendium and try to solve the following exercises.

EXERCISE 1

Fill the calculator hopscotch with the missing numbers and symbols

EXERCISE 2

Which squares should you jump into to solve the following equations?

5 + 3 = _____

6 + 8 = _____

9 + 1 = _____

EXERCISE 3

Which squares should you jump into to form an equation to the following answers?

_____ = 5

_____ = 12

_____ = 16

EXERCISE 4

Find eight equations that are equal the number 8?

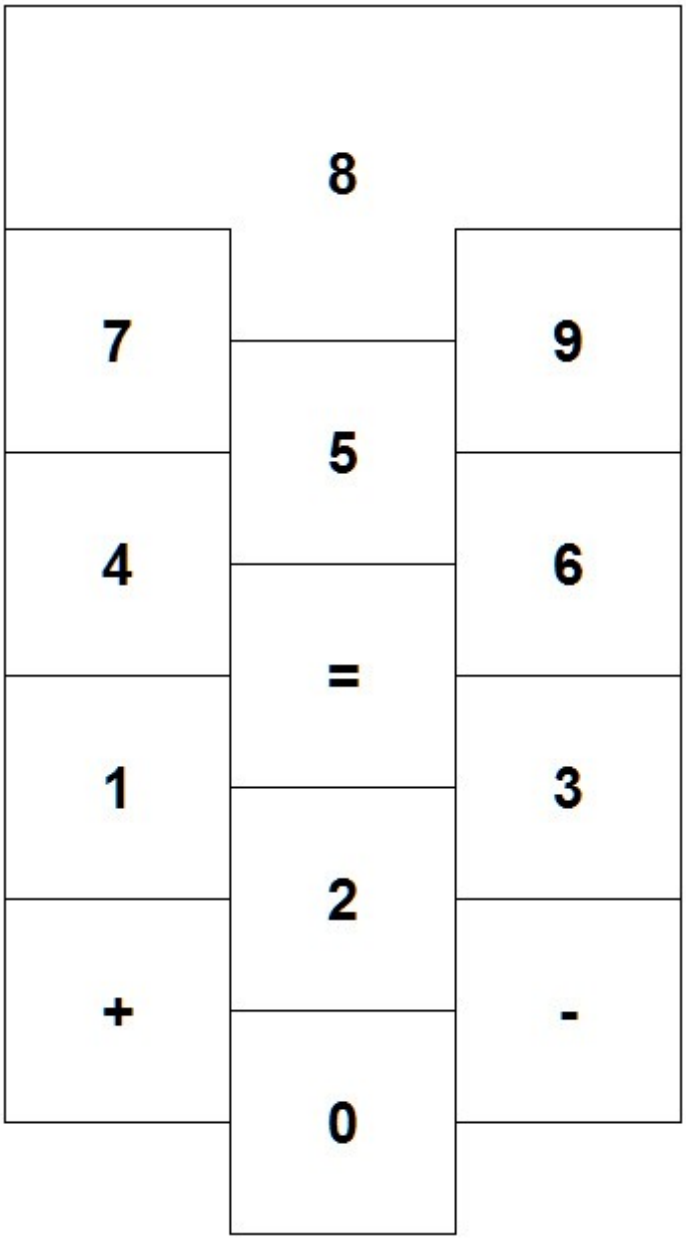
_____

_____

_____

_____

WORKSHEET 6.3 (TEACHER)



INFORMATION

Use the calculator hopscotch as described in the Compendium and try to solve the following exercises.

EXERCISE 1

Fill the calculator hopscotch with the missing numbers and symbols

EXERCISE 2

Which squares should you jump into to solve the following equations?

5 + 3 = 8

6 + 8 = 14 (one foot 1, one foot 4)

9 + 1 = 10

EXERCISE 3

Which squares should you jump into to form an equation to the following answers?

2 + 3 or 6 - 1 = 5

8 + 4 or 9 + 3 = 12

9+7 = 16

EXERCISE 4

Find eight equations that are equal the number 8?

5 + 3 = 8; 6 + 2 = 8

2 + 4 + 2 = 8; 7 + 1 = 8

9 – 1 = 8; 4 + 3 + 1 = 8

3 + 2 + 3 = 8; 6 + 4 – 2 = 8

7.1 MAGIC SQUARE (PAPER-PENCIL GAME)

MAGIC SQUARE 3X3

OBJECTIVES

- Know single-digit numbers up to 20;
- Compare and order numbers up to 20;
- Add and subtract numbers up to 20;
- Understand and use horizontal, longitudinal and diagonal concepts

TOOLS, MATERIALS AND ORGANISATION

- Material: papers, ruler, pencil;;
- Prepare 3x3 magic squares;
- Write all the numbers from 1 to 9;
- Circle the two external numbers, 1 and 9, with the blue colour;
- Circle the central number with the red colour;
- Put the number 5 in the centre of the square;
- Give more "exercises", one for each student
- Lesson takes 3 to 20 minutes

DESCRIPTION OF THE LESSON

First part of the lesson

Give to the students their magic square to complete and explain that:

- They must insert the single digit numbers from 1 to 9 in the square, and finally they will discover the magic number after having done some additions;
- Explain the horizontal, longitudinal and diagonal concepts

Second part of the lesson

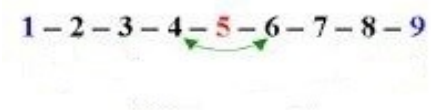
- Make the students understand which numbers they must insert in the horizontal, longitudinal and oblique lines, that give the same number, the magic number, as the result;
- The students work with numbers from 1 to 9 on the square, starting with the number 5;
- The students add numbers and discover the magic number.

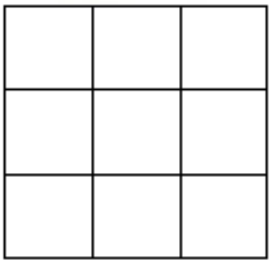
Third part of the lesson

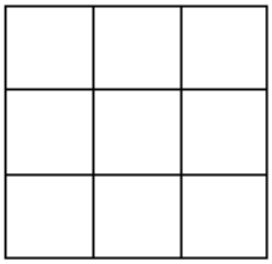
- Discover that in a 3x3 magic square, the magic number is given by the number in the center of the square, multiplied by 3;
- Once the magic number is discovered, the students must find all the possible combinations to get it, adding the numbers in the rows, lines and diagonals;
- When the students understand the rule, they can build other magic squares.

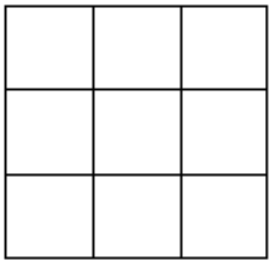
WORKSHEET 7.1 (LEARNER)

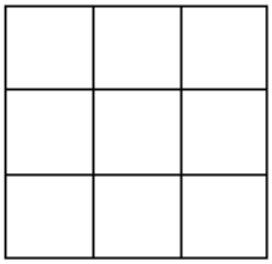
1-2-3-4-5-6-7-8-9

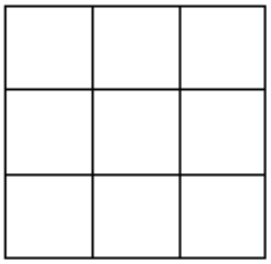












To start playing with the magic square, write all the numbers from 1 to 9 on the number line.

Which is the number in the middle? _____
Circle it in red.

Look at the numbers on the left and right side of the 5 and match pairs: one on the left, with one on the right, from the center to the external.
Write the number you have hoop in the center of the square.

Which other numbers can you write on the right and the left? Which are the available couples?

_____
Remember, you cannot separate the couples!

Let's try with the external couple 1 and 9. Make the addition: 1+5+9 is _____.

Try now in vertical. You can choose among the pairs 6 and 4, 7 and 3 or 8 and 2; choose the numbers on the right and on the left of n.5 from the center towards the border.

What pair can we put now in vertical direction?

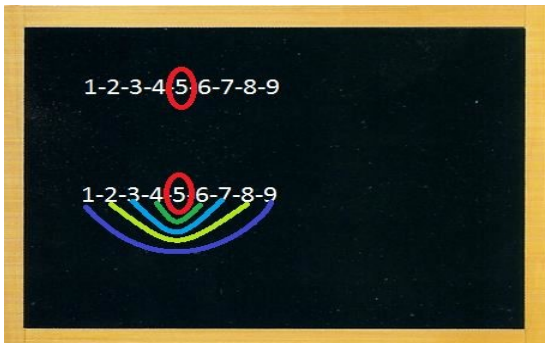
_____

At this point we must choose one pair whose sum is 10. Let's choose _____, write in vertical direction over and under the number 5. It will be __+ 5 +__ = 15 too.

Now complete the square working in the diagonal and using the other pair of numbers.
All lines have to add up to 15 in any direction.

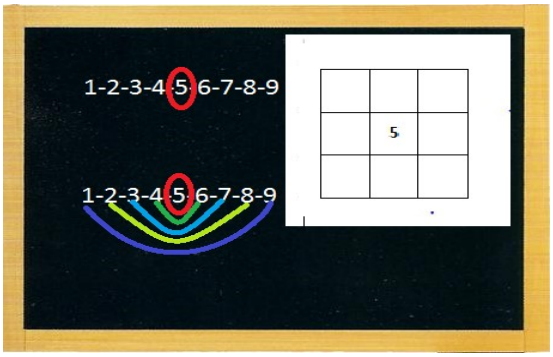
Which is the magic number? _____

WORKSHEET 7.1 (TEACHER)



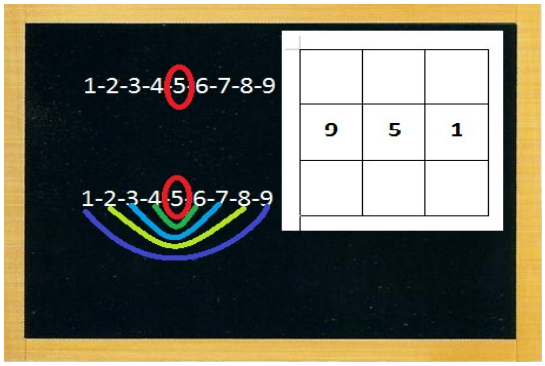
To start the game of magic square 3x3, the teacher will give a worksheet with the squares to each student.
Then he writes on the blackboard the number line from 1 to 9.

He has to circle the number in the middle (***5***) in red and matches the pairs of numbers on the left and right of the 5 with different colors.



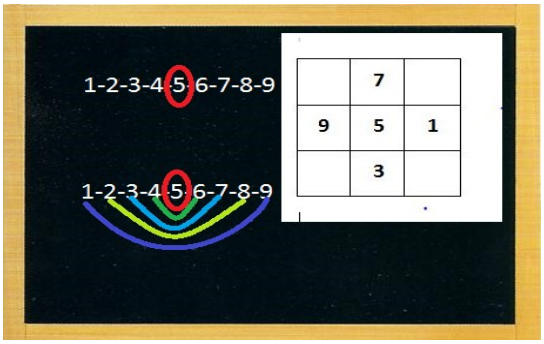
Now the students can work on their worksheet.
Write number 5 in the center.

Ask the students, which other numbers we can write in horizontal, on the left and on the right.
We have the couples: ***1-9; 2-8; 3-7; 4-6***.
Let's try with 1 and 9.
The total is ***1+5+9= 15***
We have found the magic number: ***15***

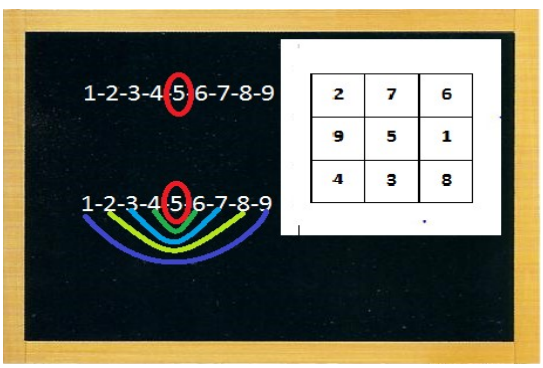


Try now in the vertical, choose among the pairs 6 and 4, 7 and 3, 8 and 2 from the centre toward the external.

Now ask:
Which pair can we put now in vertical?
Let's try with 7 and 3 because we need a pair whose total is 10.
We will write 7 and 3 in vertical. The sum is ***7+5+3=15*** again.



Let's complete the square using the other pairs.
Which number do we write?
Students will suggest 2 and 8. ***2+5+8=15***



The last pair, 6 and 4 is left; we will write in the diagonal and the square is completed. The magic numbers is ***15***!

7.2 Four Seasons (Board Game)

Objectives:

- Recognize and name two dimensional shapes (circle)
- Understand symmetry in shapes
- Understand concepts of horizontal and longitudinal lines.
- Participants should be taught to count reliably up to 20.
- Multiply using single-digit whole numbers
- Understand the concepts of double and triple
- Determine a next term in linear patterns (e.g., 3, 6, 9, …)

Tools, Materials and Organisation

- Build a round table each four players
- Each groups needs four series of twelve pawns of four different colours (red, green, white and black)
- Prepare copies from the worksheet for each student.
- The lesson takes 45 minutes.

Description of the Lesson

First part of the lesson

- Give the students an example of the round board and teach them how to build it showing a video https://www.youtube.com/watch?v=CCFDhtg-G4U&feature=youtu.be

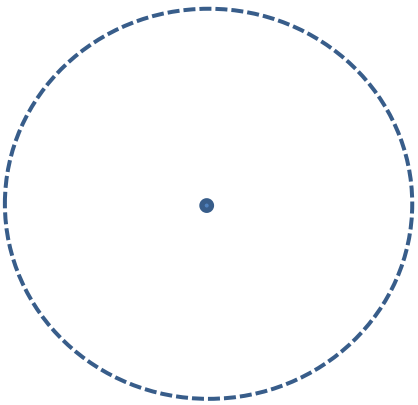
Second part of the lesson

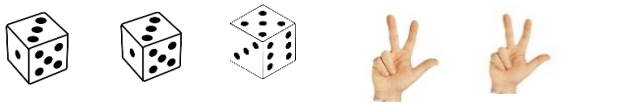
- Build up groups with 4 persons and explain the game
- Hand out the worksheets to each person to be filled in during the game.
- The participants learn to count counting all the pawns.
- The students learn to add quantities.
- The participants learn that the mental calculation is strategic and flexible; it is guided by the estimation of the possible result.
- They practice the game and learn the meaning of double, triple and scale.

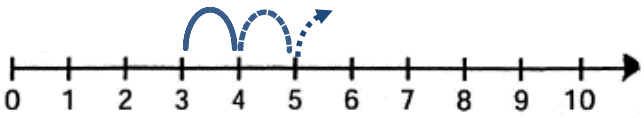
Useful Hints

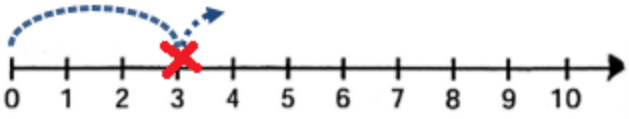
- At the end of the lesson the worksheet is completed.
- If the participants cannot read, the teacher has to guide them.
- If participants have difficulties to count or do the addiction, they have to train – they need more time.
- If participants have difficulties to count, the teacher has to divide the lesson into two lessons: First lesson: play and count by mind, second lesson: play and learn the strategies.
- Following lesson: practice game strategies and problem solving

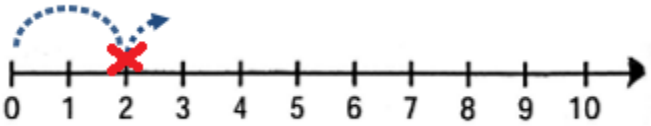
WORKSHEET 7.2 (LEARNER)











Section A - Basic concepts of geometry

Construct a circle

Draw lines from the centre to the border: Are the lines of the same length?

_____

Draw a line passing through the centre and split the shape into two parts.

Fold the paper along the line. Are the two parts identical? _____

Fold the paper in the centre again: the line has to cross the other line.

Draw a line along the folds. How many parts has the circle been divided into?

_____

You have two lines: one is straight up and down the other one goes left-to-right. What are they called?

_____

Section B – Start the game

If the dice roll gives two identical numbers in each group, two from each group show the numbers (3) with your fingers.

Show the quantity of the two dice on the number line. Which is the last number?

_____

Add the fingers. How many? _____

Now put a cross representing the numbers of the dice: 3, 6. Then draw a line.

What is a number repeated twice called?

_____

Continue on the line and determine the next rhythm pattern:

2, ___, ____, ____, ____.

What is a number which is repeated three times called?

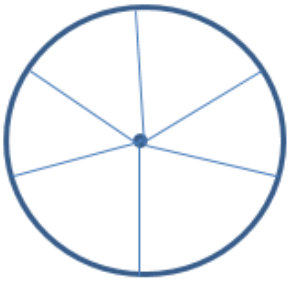
_____

WORKSHEET 7.2 PAGE 1 (TEACHER)

Section A - Basic concepts of geometry

Give the students an example of the circle and tell them they have constructed a new one. Show them the video.

Ask the students to draw lines from the centre to the border and make them measure the lines.



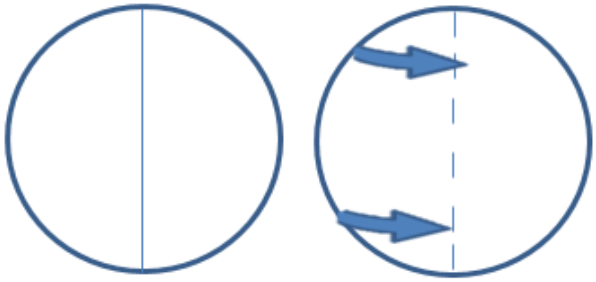
What do the pupils realize?

The pupils realize that they have all the same length!

Ask the students to draw a line passing through the centre and split the shape into two parts folding the paper along the line.

Compare both parts!

Both parts are identical.

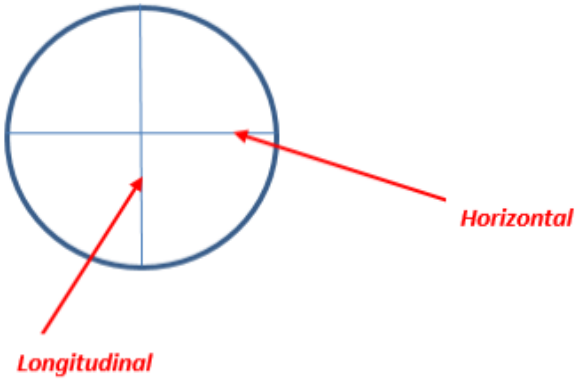


Ask the students to fold the paper in half again: the line has to cross the other line, draw a line along the fold.

How many parts has been the circle divided into?

Four

Make the students put the paper so that the lines are one longitudinal and the other one horizontal. Check the position of the lines.



Explain that the straight up and down direction is called “longitudinal or vertical” and the left–to-right one is called “horizontal”.

vertical

WORKSHEET 7.2 PAGE 2 (TEACHER)

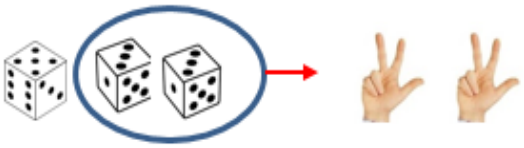
Section B – Start the game

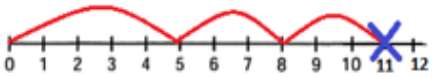
Introduce the concept of double and triple by rolling dice.

Make the students represent single digit numbers using the fingers and the number line.

Represent on the number line the numbers on the dice one by one.

Let's focus on the final number. It is the total of the addition.





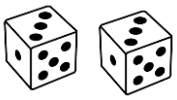
Now make the same with the fingers. Count them.

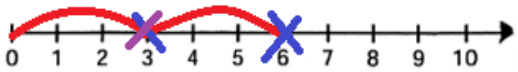
5+3+3 = 11

Is the total the same? *Yes*

Represent directly the numbers of the dice on the number line: 3 and the other 3.

In this case let them use a cross to mark the numbers.





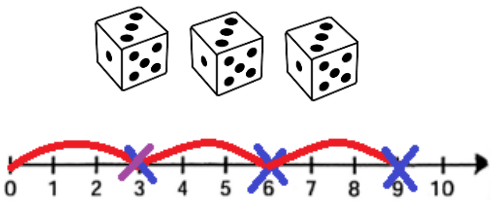
Tell them that a number repeated twice is called "double".

Make them practice double with other numbers (using their fingers):

2 → 4; 4 → 8…

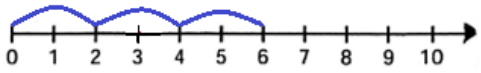
Can they recognise double?

WORKSHEET 7.2 PAGE 3 (TEACHER)



Make the students do it with three dice:

3, 3, 3 jumping on the number line



Teach them that a number repeated three times is called “triple”.

Make them practice with 2!

What is the triple of 2? ***6***

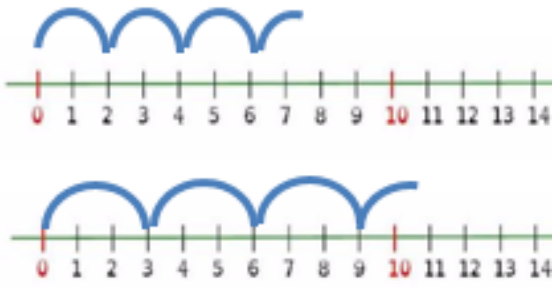
What is the triple of 3? ***9***

Now draw on the blackboard a longer number line to practice with higher numbers.

What is the double of 7? ***14***

What is the triple of 4? ***12***

What is the triple of 5? ***15***



Make the students determine the next rhythm pattern in the number line.

Try 2 by 2

Try 3 by 3

within the number 20 on the line

7.3 Steal the Pile (Card Game)

Objectives

- Participants should be taught to count reliably up to 40.
- Order and compare numbers up to 40.
- Add single-digit numbers with totals to 10.
- Develop easy strategy skills Sort and classify objects using a single criterion.
- Understand probability.
- Identify the range of possible outcome when using a card
- Determine the approach, materials and strategies.
- Develop the mental calculation abilities

Tools, Materials and Organisation

- Take a deck of cards for each 2 - 4 players.
- Prepare copies from the worksheet for each student.
- The lesson takes 45 minutes.

Description of the Lesson

First part of the lesson

- Explain the game “Steal the pile”
- Build up groups with 3 or 5 persons: 2 or 4 are playing; one more is watching the game.
- The participants play several times the game “Steal the pile”. This makes them feel self-confident.
- Follow up: learn the value of the cards in the deck

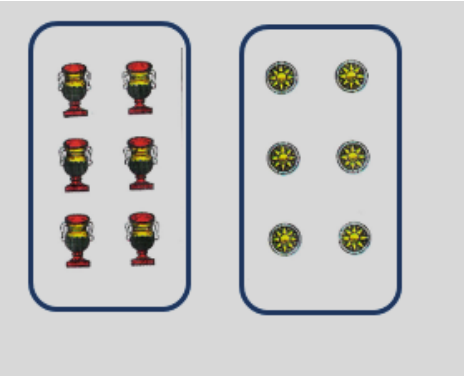
Second part of the lesson

- The participants learn to count up to 40 by counting all the cards.
- The participants learn to recognize the cards, the suit of marks, and the face cards.
- Students practice the game to learn how to calculate with subitizing small amounts through the cards
- They add and remove a quantity using the subitizing
- They learn to use strategy to add from the major number
- The participants learn that the mental calculation is strategic and flexible; it is guided by the estimation of the possible result.
- Hand out the worksheets to each person.
- Follow the instruction on the worksheet

Useful Hints

- At the end of the lesson the worksheet is completed.
- If the participants cannot read, the teacher has to guide them.
- If participants have difficulties to count or do the subitizing, they have to train – they need more time.
- If participants have difficulties to count, the teacher has to divide the lesson into two lessons: First lesson: play and count by mind, second lesson: play and learn the subitizing.
- Following lesson: Look for another game, in which the participants have to count up to 10 or up to 40.

WORKSHEET 7.3 (LEARNER)



Count all the cards.

The cards are___ and highest value is_____

Do you remember all the cards one by one?

Can you write the value of these two cards?

_____.

If your answer is yes, please write the value of these cards here: _____

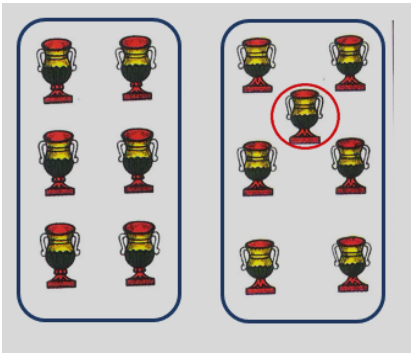
Do you remember how many suit marks there are in a deck of cards? _____

Write the name and the correspondent value of the three cards that you see on the left.

_____

_____

_____



Now try to describe the difference between the two cards that you see on your left

_____

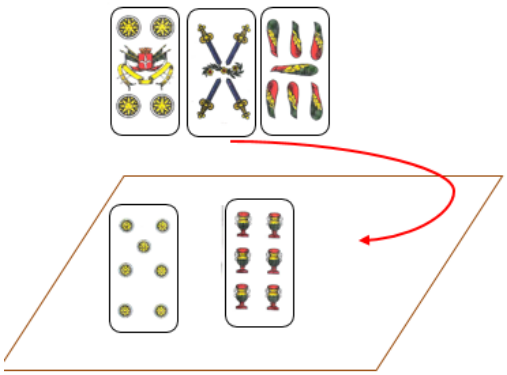
_____

Now if you have on the table seven and two and you hold in your hand a horse can you take all the cards?

_____

If your answer is yes, please explain why

_____



Look at the cards on your left. Imagine having them in your hand:
You have in your hand two ___ and one ___
Cards on the table are ___ and ___

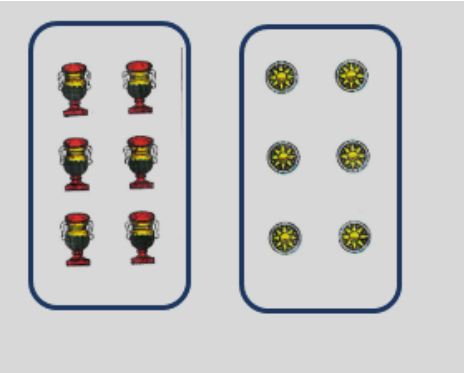
Write please your next move and explain why:

_____

_____

_____

WORKSHEET 7.3 (TEACHER)



Count all the cards.

The cards are _40_ and highest value is_10_

Do you remember all the cards one by one?

Yes

Can you write the value of these two cards?

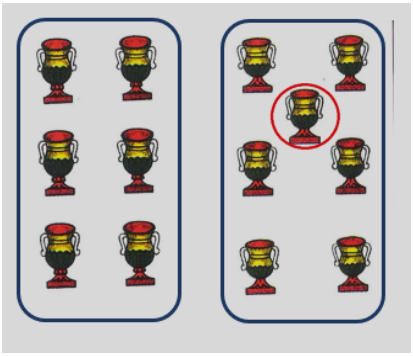
Yes

If your answer is yes, please write the value of these cards here: *6 and 6*

Do you remember how many suit there are marks in a deck of cards? *Yes*

Write the name and the correspondent value of the three cards that you see on the left.

Infantry man = 8, Horse = 9, King = 10



Now try to describe the difference between the two cards that you see on your left

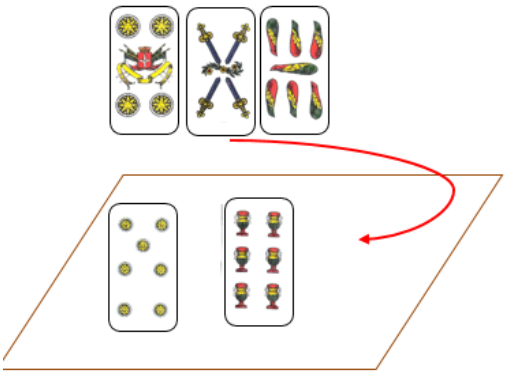
The second one has one more cup in the middle: they are a 6 and a 7

Now if you have on the table seven and two and you hold in your hand a horse can you take all the cards?

Yes

If your answer is yes, please explain why

Because 7 + 2 is 9 and the horse value is 9



Look at the cards on your left. Imagine having them in your hand:

You have in your hand two _4_ and one _7_

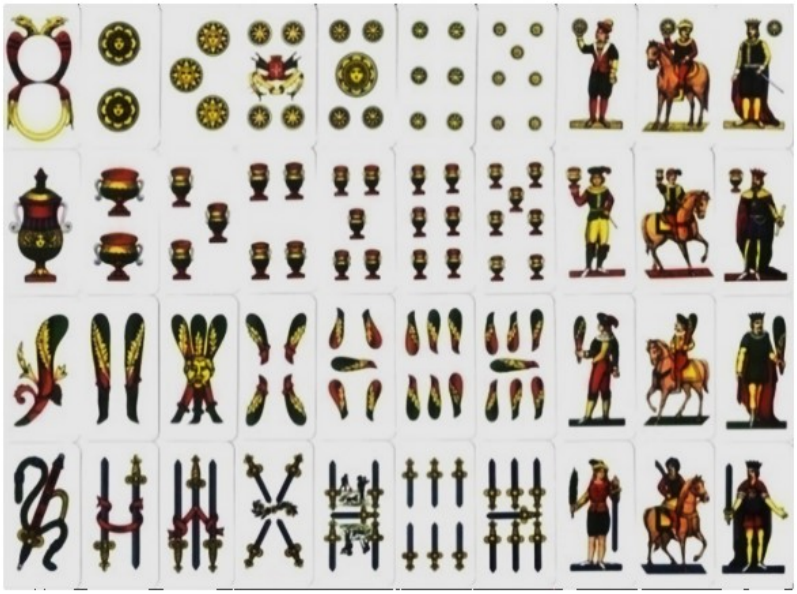
Cards on the table are _7_ and _6_

Write please your next move and explain why:

I do not play 7, but 4

If my opponent has a 4 too, and takes mine, I will steal the pile with my second 4

HINTS FOR THE TEACHER - FOLLOW UP



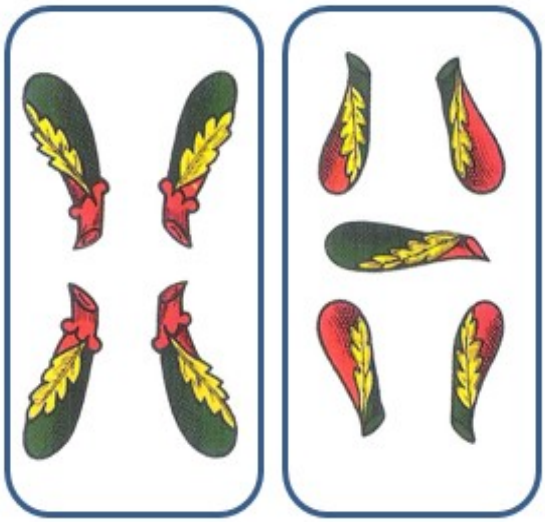
Recognize the cards

- Order them in a line according to the increasing value from 1 up to the king (10)
- Recognize all the numbers in the different suit marks
- The highest value is 10!
- Recognize the three main figures and their own value.

King=10

Horse=9

Infantry-man=8



Then you can start the game showing to the participants the 40 cards and the different suit marks.

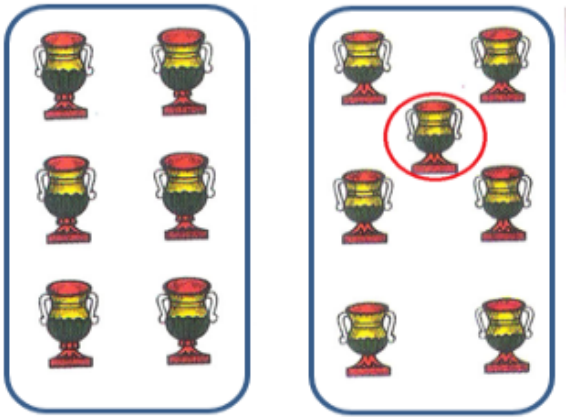
They must recognize all the cards. Do they remember them?

Count the 40 cards one by one until they have learnt the numbers.

Give them three numbers to be put in increasing order. E. g. 7, 9, 3

Which is the correct order?

3, 7, 9 (mental skill)



Play the game

Groups of two or four participants play the game. The participants play the game "Steal the pile" with open cards.

You can guide them in the phases of the game: Each player picks up his hand and compares the cards he is holding, with those on the table, looking for matches

You support the identification of the ordered configuration of the objects in the cards, connecting them to the corresponding amount.

This can facilitate the calculation skills (subitizing). In this case, for example, the teachers will focus on the position of the elements (cups). They are in 2 lines and it is easy to memorize the number 6. In the second card, it compares one more not lined-up element. It means that it is enough to add 1 to the memorized card (+1).

Adding numbers

The player can take more cards, where the sum is the same number he holds. In this case he has to add the numbers of the cards on the table (see Math-GAMES Compendium)

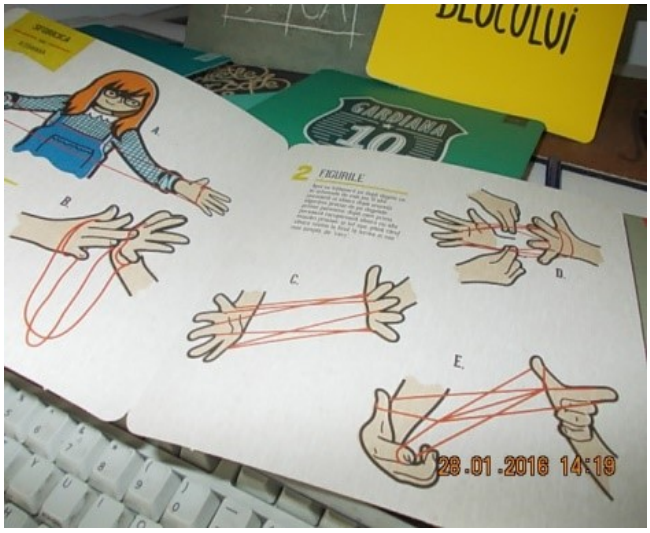
Counting

The player counts the items using his fingers. Later the player learns to count starting directly from the major number.

Strategy

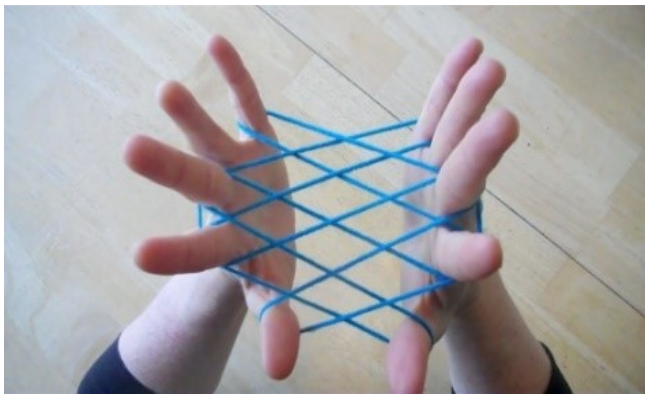
You can show to the players a very simple strategy: If a player holds, for example, a seven and two fours and on the table there is a seven, it may be convenient not to play the seven but the four. If the opponent has a four too and takes it, the first can steal the pile with his second four.

8.1 Crows Feet - Cat's Cradle (String Figures)



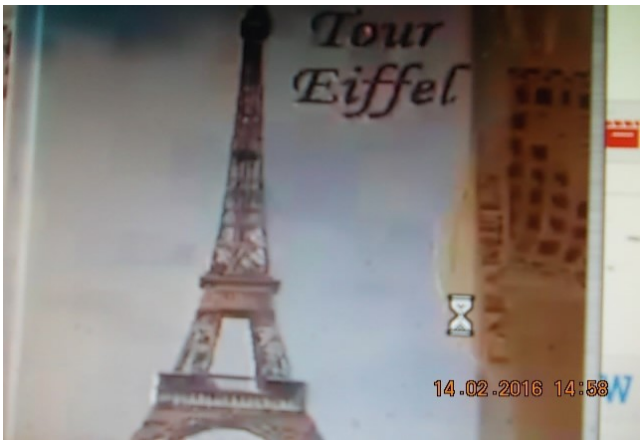
Objectives

- The participants learn to do *Crow's Feet, Cat's Cradle, or ….*
  The students learn the following geometric figures: angle, circle, line, rhomb, rectangle, parallel lines, triangle, and square.
- The players learn to count reliably up to 20 items.
- This game for two players is extremely easy, but in order to win the game the players need abilities, intelligence, patience, concentration and calmness.
- Playing this game the players socialize, develop their abilities of space orientation, learn to distinguish among geometrical figures, improve their knowledge of geography and general culture (e.g. the Tour Eiffel variant).



Tools, Materials and Organisation

- This game can be played outdoors, indoors, at school, during the break;
- It can also be played by the adults who want to teach their children a new game.
- It is played strings in hands.



Description of the Lesson

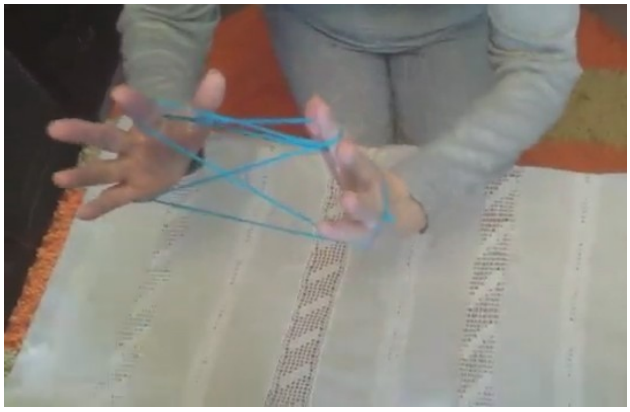
First part of the lesson (5 minutes):

- Explain the game (the participants already know the rules of the game "The Crow's Feet" or the "Little String" (Are there any other names for this game? Maybe "The little string", "The little thread", "The cat's ladder")

Second part of the lesson (20 minutes):

- Build up groups of 2 persons (3-4 groups)
- Stage 1: They take a 70 cm long piece of string and tie its ends (in case there are 3-4 teams each team takes a string of a different colour: red, blue, green, etc.)
- Stage 2: The 1st player coils it round 2 fingers.
- Stage 3: The second player takes over the string with his fingers using certain precise movements.
- Stage 4: The 1st player recovers the string with his own fingers.

- Stage n: The stages are repeated until the end when the string must unwind.
- End of the game: It is played in the initial circle form.

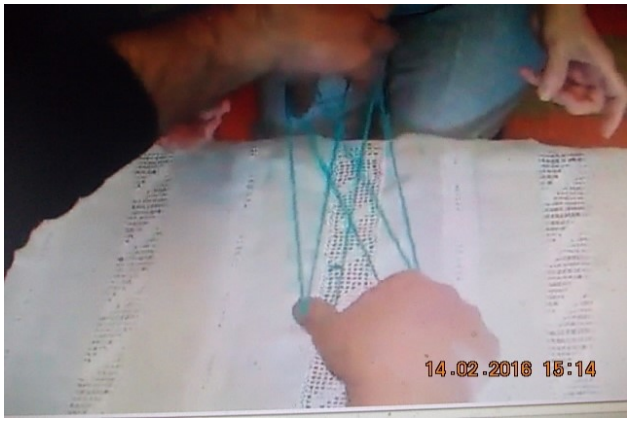


Third part of the lesson (15 minutes):

- Hand out the worksheets (one to each person).
- Follow the instructions on the worksheet.
- Mathematical exercises using tables with definitions and geometrical figures

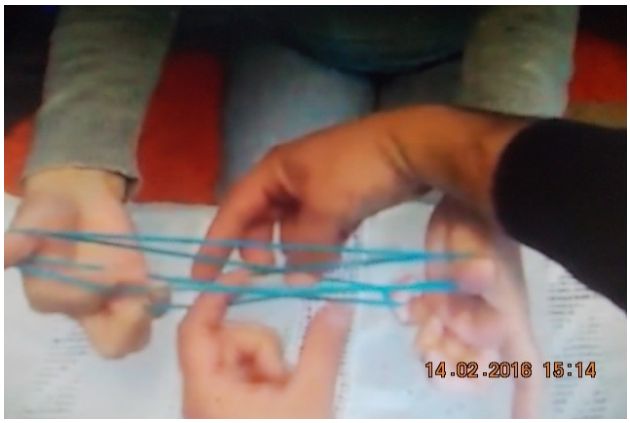
The last part of the lesson (5 minutes)

- Each team or participant hands out its/his/her achievement and presents it to the others.
- The winner, team or person is appointed.



USEFUL HINTS

- Be a playful teacher, the participants appreciate the teachers who take part in their game;
- It is very important, that the teacher should be aware of the students' abilities;
- Unveil certain tricks of this game which you have already practised with other players in other lessons;
- Use a game lexis appealing to your childhood memory: „Outch! Don't pull, I've caught my fingers!''
- This game involves both players and spectators; they all want to learn how to count, to visualize geometrical figures and how to get acquainted with something new.
- The game enhances their interest in being creative, in discovering things from simple to complex levels, in orientation and coordination.



- The game creates an atmosphere of joy, friendship and social involvement connecting different generations.

EXAMPLES, REFERENCES AND LINKS

https://en.wikipedia.org/w/index.php?title=File:String_Figures_and_How_to_Make_Them.djvu&page=1

https://www.youtube.com/channel/UCvuYRVDPNWRNO5SwQiRre4g

https://www.youtube.com/watch?v=zIHfMkxVInU

https://www.youtube.com/watch?v=o-ekqfSz428

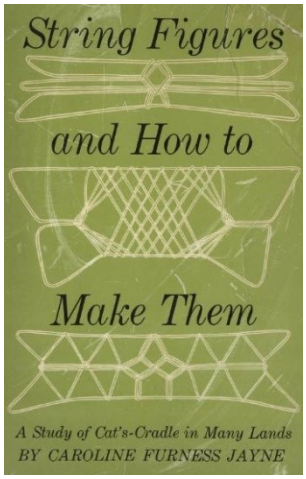
http://www.infatablocului.ro/

http://www.wikihow.com/Do-String-Figures
https://youtu.be/AIIAZz37dYQ

https://youtu.be/FyTi7Pf7LXk

https://youtu.be/KNDErjr2p6c

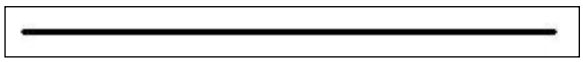
https://youtu.be/Vb6DWj4OLd0



WORKSHEET 8.1 (LEARNER)

Read and memorize the **definitions** of the geometrical figures used in the game.

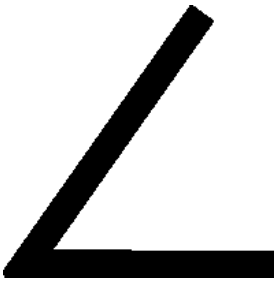
Straight Line



1. A simple and continuous touch of a pen, chalk or pencil (in the shape of a thread) drawn on a surface.

2. (Mathematics) A trajectory described by a material point in a continuous movement or by the intersection of two surfaces.

Angle



A configuration of two line segments meeting at a point.

Question 1

What geometrical figures are presented in the Crow's feet game?

Triangle

Spiral (curved) line

Rectangle

Correct answer: _____

Question 2

Which geometrical figure is associated with the end of the game?

Angle

Circle

Line

Correct answer: _____

Question 3

Draw a geometrical figure that you have discovered in this game using a coloured pencil.

Question 4

How many fingers do the players use in this game?

2

10

20

Correct answer: _____

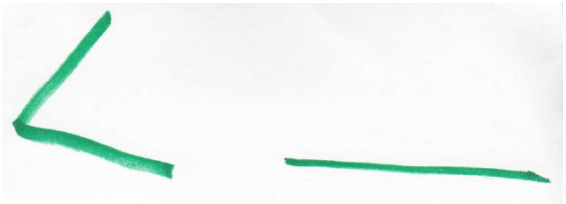
Solution to the questions:

Question 1: Triangle and Rectangle

Question 2: Circle

Question 3: see right

Question 4: 20



8.2 Skipping Rope (Outside)

Objectives

- **They come to count**, add and subtract single-digit numbers up to 10 (forwards, from 1 to 10 and backwards, from 10 to 1).
- **They recognize** and name two dimensional shapes (circle, semicircle)
- The game implies imaginative memory, high speed, dexterity and ability in creating authentic shows.
- **Skills required:** orientation abilities, counting while jumping, and strategically change.
- The game generates social harmony between generations: players enjoy it from childhood to adult life.

Tools, Materials and Organisation

- Outdoor games, urban, rural environment
- 4-5 at the 10-to-1 variant and at the skipping rope in 3 variants: 10-15 players
- Each team involving minimum 2 players needs a skipping rope, a good physical condition and playing abilities.
- The lesson takes 45 minutes.
- The learners can do the second part of the lesson only, if they have learned about coordinates in lesson 8.2.

Description of the Lesson

First part of the lesson (25 minutes)

- Make groups of two participants. Play the game several times. Discuss about the strategies.

The teacher and an experienced participant describe and illustrate the stages of the game:

- Stage 1: It starts from 10 skips, performed according to each player's choice
- Stage 2: 9 skips, jumping alternatively with the right leg, with the left leg
- Stage 3: 8 skips, the players jump on two legs, kept close
- Stage 4: 7 skips on one leg, at the player's choice
- Stage 5: 6 skips keeping the legs straight and close, first pushing the left forwards and the right behind and then the other way round
- Stage 6: 5 skips alternating open and close legs
- Stage 7: 4 skips, 2 times on the right leg and two times on the left leg
- Stage 8: 3 skips with crossed legs
- Stage 9: 2 skips on one leg, at one of them crossing the arms
- Stage 10: 1 skips with crossed arms.

Second part of the lesson (10 minutes)

- The worksheets are handed out to each person.
- They follow the instructions on the worksheet.
- If there is enough time, they can talk about differences between two or three versions of the game description 8.2 (Skipping rope in 3 and The Clock variant).

The last part of the lesson (10 minutes)

- The best rope skipper is appointed.
- The best exercises of counting skips accompanied by the most agreeable lexis, the most beautiful and the most difficult moments of the game are mentioned again.
- The teacher and his assistant check the mathematical abilities studied while playing the game by means of worksheets and 3 exercises.
- The best rope skipper will give a short show.

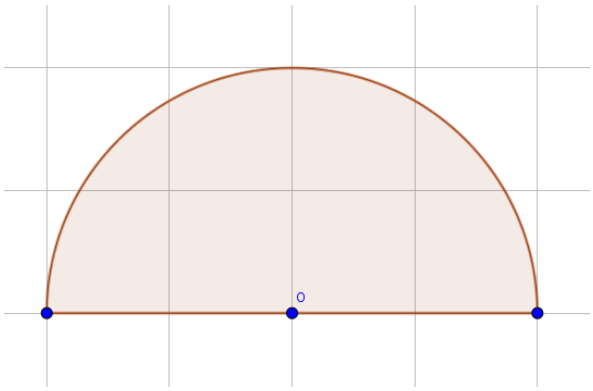
Useful Hints

- Do not jump too high, but just as high as to allow the rope to pass under your feet. Use your wrists to move the rope, not the shoulders while keeping the elbows close to the body.
- There are variants in which at the stage 5 (6 jumps), the player says: „green leaf wait for 6, /I've done 6!"(In Romanian this distich rhymes and it has a mnemonic function).
- Other specific vocabulary elements at the Clock are: You failed / Fail / Miss", "I was at 5! You were not you were at 7! Give me the rope back, I go home!"
- Alternate the spinning directions lest you should feel giddy when practising the „Clock" variant.

WORKSHEET 8.2 (LEARNER)

Information Semicircle

A geometrical Figure formed from half of the circle is called semicircle.



Question 1

What geometrical Figure is in this image of a rainbow?

Circle
Square
Semicircle

Answer: _____

Question 2

What connection exists between the Rainbow and a Skipping Rope?

Answer _____

Question 3

What is a semicircle?

A semicircle is a _____

Question 4

Draw a semicircle using a coloured pencil or a marker (like the example on the left):

Question 5

Count the rope skips from 1 to 10 and write it down:

1__2_____

Question 6

Count the rope skips from 10 to 1 and write it down:

_10___9_____

Question 7

You win the game "Skipping Rope",

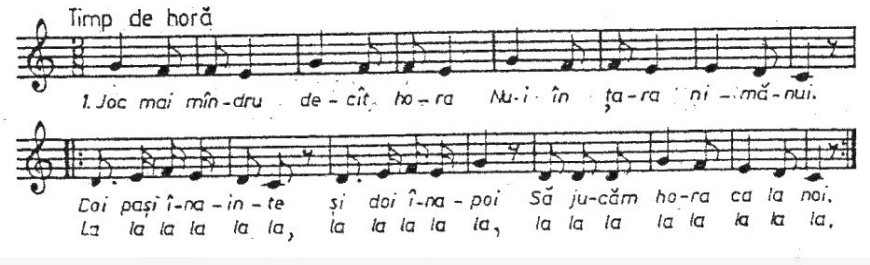
1. if you did all the skips from 1 to 10 well
2. if you made the least number of mistakes
3. if skipping from 10 to 1, you were the first player to reach 1

Answer _____

Solution to the questions:

Question 1: Semicircle
Question 2: The shapes of rainbows and skipping ropes are semicircles.
Question 3: A semicircle is half of a circle.
Question 5: 1 2 3 4 5 6 7 8 9 10
Question 6: 10 9 8 7 6 5 4 3 2 1
Question 7: You win the game, if you are the first player, who reaches 1 (Answer 3)

8.3 HORA (ROMANIAN DANCE)



OBJECTIVES

- Understand a coordinate system.
- Count reliably up to 10 (numeracy).
- Know the follower and the predecessor of a one-digit number.
- Recognize and name two-dimensional shapes (geometry: spiral, circle).
- Reintroduce the old Romanian traditional dance to the young generation
- Bind again the links between the old and the young generations.

TOOLS, MATERIALS AND ORGANISATION

- The lesson lasts for 45 minutes.
- It requires a sufficiently great number of attendants (15-20 or more).
- Worksheets are prepared.
- The hora can be danced in the classroom, in the yard, in the garden, in the field, in a village, in a special hora place.
- Internet source, dvd, film etc.
- The participants need basic notions of rhythm, music and dance steps

DESCRIPTION OF THE LESSON

First part of the lesson (35 minutes)

- Free space for dancing.
- Danced by more persons in a circle.
- Play the game several times. Discuss about the strategies.
- The participants watch examples of traditional Hora recorded on a DVD.
- The attendants are invited to stand in a big circle similar to what they had seen in the film.

Second part of the lesson (10 minutes)

- Hand out the worksheets to each person one
- Follow the instruction on the worksheet

USEFUL HINTS

- Try to tune up fast to the rhythm and dance steps
- Practise some traditional hollers
- Memorize the music on which you dance
- Read Math-games Compendium 8.3 in advance
- Become a promoter of this dance in your community
- Pay attention to the Math elements of the Hora

http://www.bistritaculturala.ro/stire.php?id=39

https://ro.wikipedia.org/wiki/List%C4%83_de_dansuri_populare_rom%C3%A2ne%C8%99ti

http://www.juniisibiului.ro/

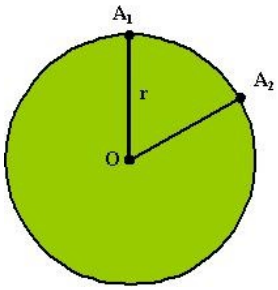
http://www.infotravelromania.ro/fotografii_traditiiromanesti.html

https://www.youtube.com/watch?v=oVrVjfYJKSc

WORKSHEET 8.3 (LEARNER)

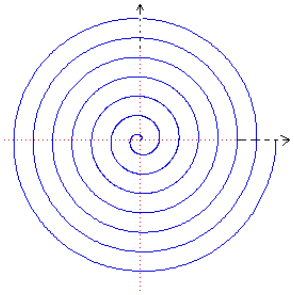
Circle

Definition: A circle is the geometric locus of all the points of a plan situated at the same distance r from the point O, named the circle center.



Spiral

Definition:
a curved line which departs from the centre in its spinning movement.



Spiral in the nature: Ammonite

Question 1

Draw a circle and a spiral using a marker or a colored pencil:

Question 2

The figure of a Hora dance is a

- Circle
- Square
- Triangle

Answer: _____

Question 3

How many dancers can dance Hora?

- 1 to 10.
- From several people to 20
- From several to a lot of people

Answer: _____

Question 4

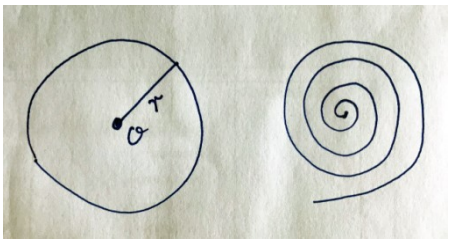
What elements can be found in the Hora dance?

- Music
- Rhythm
- Movement

Answer: _____

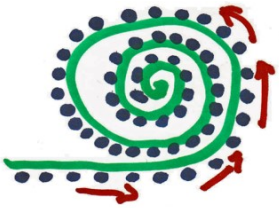
Solution to the questions:

Question 1: see right
Question 2: Circle
Question 3: From several to a lot of people
Question 4: Music, rhythm and movement

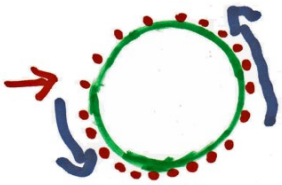


WORKSHEET 8.3 (TEACHER)

1. PHASES OF HORA (TRADITIONAL)



a) Spiral entry into the dance (Beginning of the Hora)

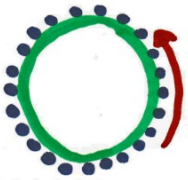


b) Circle of the Hora

1. Three steps left

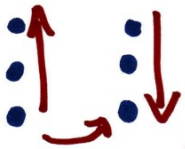
2. Three steps right

3. Two beats (step dance)

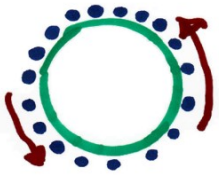


4. Spin Hora

2. VARIANT OF THE HORA (BASIC)

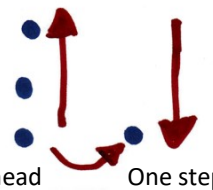


Three steps ahead Three steps backwards

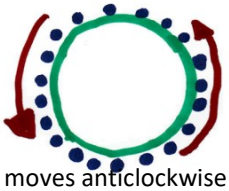


The circle of Hora moves anticlockwise to the right.

3. VARIANTS OF THE HORA (THE BIG HORA)



Three steps ahead One step backwards



The circle of Hora moves anticlockwise to the right.

9.1 Fifteen-Game (Board Game)

Objectives

- Count, read, and write whole numbers up to 20
- Order and compare numbers up to 20
- Recognize, describe, and extend patterns and determine a next term in linear patterns
- Recognize and name two dimensional shapes
- Describe length and width of shapes
- Measure areas by counting squares

Tools, Materials and Organisation

- A sliding 15-puzzle consisting in a 4x4 square box and fifteen pieces numbered from 1 to 15.
- A sheet of patterns that each student should try to solve.
- The lesson takes two sessions of 45 minutes.

Description of the Lesson

First part of the lesson:

- Arrange groups with 2 persons.
- Give them an empty square box and fifteen numbered cubes.
- Tell them to place the cubes in ascending and descending order out of the box.
- Ask them to puts the cubes into the box in ascending and descending order in rows.
- Ask them to put the cubes into the box in ascending and descending order in columns.

Second part of the lesson:

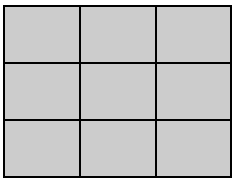
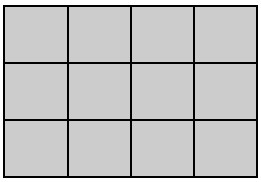
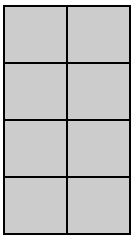
- The teacher explains the game
- One of the students is playing the game while the other one is watching.
- One of the students (the observer) put the cubes in the box in random order.
- The teacher gives them a sheet with 6 different patterns.
- The other student (the player) has to place the pieces in order by making sliding moves that use the empty space.
- The participants play several times the game changing roles (player and observer) trying to solve most of the patterns in the sheet.

Useful Hints

- Make sure the students can count well and order the numbers in ascending and descending order.
- Make sure the students understand the concept of row and column.
- Use the first part of the lesson to explain and clarify these items. They must understand the underlying concepts; otherwise they won't be able to play the game.

WORKSHEET 9.1 (LEARNER)

How many <u>rows</u> are there in each figure? How many <u>columns</u> are there in each figure?

A	B	C
		
Rows:	Rows:	Rows:
Columns:	Columns:	Columns:

Order the following numbers from lowest to highest:

6 – 1 – 5 – 10 – 9 – 3 – 8 – 15

9 – 4 – 8 – 13 – 12 – 2 – 6 – 7

Order the following numbers from highest to lowest:

8 – 14 – 5 – 7 – 10 – 3 – 11 – 2

12 – 5 – 10 – 7 – 3 – 4 – 6 – 13

Fill in the blanks at the tables with the suitable numbers to complete the sequences properly:

1	2		4
	6	7	8
9		11	12
13	14		

15	11	7	3
14		6	
	9	5	1
12	8		

4	5	12	13
3	6		
	7	10	15
1		9	

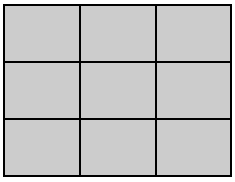
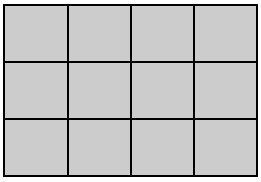
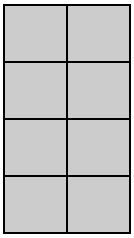
1	5	9	
2	6		14
3		11	15
	8	12	

1		3	4
12	13		5
		15	6
10	9	8	7

7		9	
6	15		11
	14	13	
4	3	2	1

WORKSHEET 9.1 PAGE 1 (TEACHER)

How many rows are there in each figure? How many columns are there in each figure?

A	B	C
		
Rows: *3*	Rows: *3*	Rows: *4*
Columns: *3*	Columns: *4*	Columns: *2*

Order the following numbers from lowest to highest:

6 – 1 – 5 – 10 – 9 – 3 – 8 – 15

1	*3*	*5*	*6*	*8*	*9*	*10*	*15*

9 – 4 – 8 – 13 – 12 – 2 – 6 – 7

2	*4*	*6*	*7*	*8*	*9*	*12*	*13*

Order the following numbers from highest to lowest:

8 – 14 – 5 – 7 – 10 – 3 – 11 – 2

14	*11*	*10*	*8*	*7*	*5*	*3*	*2*

12 – 5 – 10 – 7 – 3 – 4 – 6 – 13

13	*12*	*10*	*7*	*6*	*5*	*4*	*3*

Fill in the blanks at the tables with the suitable numbers to complete the sequences properly:

1	2	*3*	4
5	6	7	8
9	*10*	11	12
13	14	*15*	

15	11	7	3
14	*10*	6	*2*
13	9	5	1
12	8	*4*	

4	5	12	13
3	6	*11*	*14*
2	7	10	15
1	*8*	9	

1	5	9	*13*
2	6	*10*	14
3	*7*	11	15
4	8	12	

1	*2*	3	*4*
12	13	*14*	5
11		15	6
10	9	8	7

7	*8*	9	*10*
6	15		11
5	14	13	*12*
4	3	2	1

WORKSHEET 9.1 PAGE 2 (TEACHER)

Try to solve the patterns below. Remember that you must place the pieces in this order only by making sliding moves that use the empty space.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Horizontal

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	

Vertical

4	5	12	13
3	6	11	14
2	7	10	15
1	8	9	

Up and down

7	8	9	10
6	1	2	11
5	4	3	12
	15	14	13

Spiral

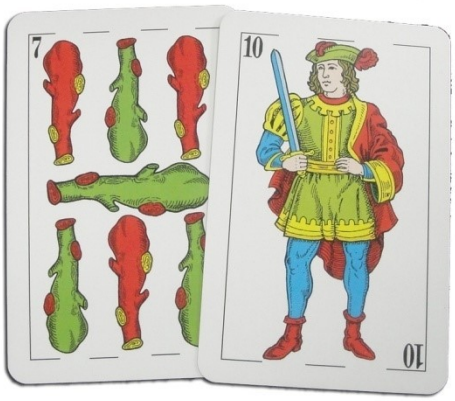
1	2	3	4
12	13	14	5
11		15	6
10	9	8	7

Peripheral

7	11	14	
4	8	12	15
2	5	9	13
1	3	6	10

Diagonal

9.2 Seven and Half (Card Game)



Objectives

- Count reliably up to 20 items.
- Read numbers up to 20.
- Add single-digit numbers with totals to 20.
- Read and add halves of quantities.
- Use a calculator to check calculation using whole numbers.
- Understand probability.
- Identify the range of possible outcome when using a card.

Tools, Materials and Organisation

- A Spanish pack of playing cards.
- The players are seated in groups not larger than 8.
- The lesson takes 45 minutes.

Description of the Lesson

First part of the lesson:

- The players are seated in groups not larger than 8.
- Give them a pack of cards and show them the value of each one.
- Practice counting different combinations of cards.

Second part of the lesson:

- Explain the game.
- Ask them to take a card from the deck. Who takes the highest number is the bank.
- The participants play the game several times until they have learnt the performance.

Useful Hints

- Participants must know well the value of the cards before they start playing.
- Students should be aware of whether to risk or not depending on the cards they own
- They could win the game even with a low value; most important is to observe the attitude of the other players.

WORKSHEET 9.2 (LEARNER)

A B C D

Who is the player with the highest value? _____

A

B

C

D

How many points do you have if you get this combination of cards?

A _____

B _____

C _____

D _____

A

B

C

D

How many points do you need to get seven and half?

A _____

B _____

C _____

D _____

A

B

C

D

What option would you try?

____ *I'd better ask for another card.*

____ *Nothing to do. I've lost the game.*

____ *I just stand. I don't want to risk.*

____ *I got seven and half. I've won.*

WORKSHEET 9.2 (TEACHER)

A B C D

Who is the player with the highest Value? ***B***

A

B

C

D

How many points do you have if you get this combination of cards?

A ***8,5***

B ***3***

C ***7,5***

D ***9***

A

B

C

D

How many points do you need to get seven and half?

A ***3,5***

B ***4,5***

C ***I have overcome the score***

D ***1***

A

B

C

D

What option would you try?

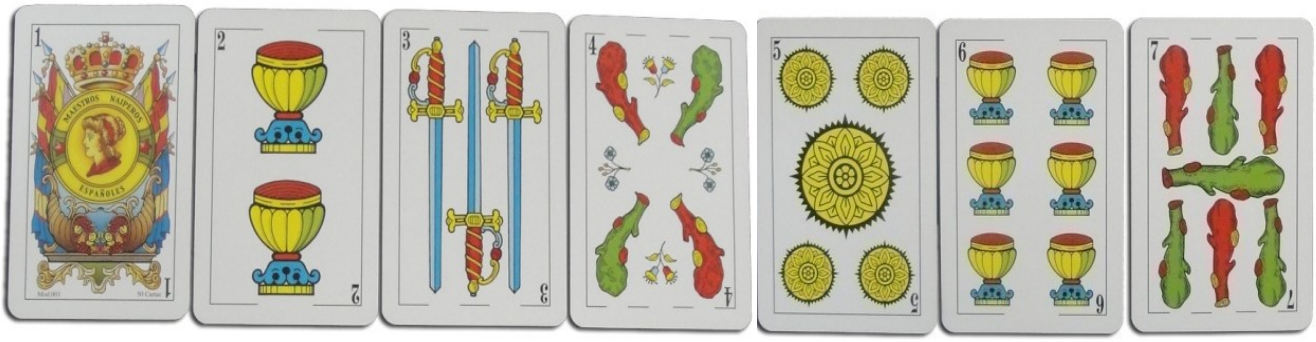
C *I'd better ask for another card.*

D *Nothing to do. I've lost the game.*

A *I just stand. I don't want to risk.*

B *I got seven and half. I've won.*

COPY TEMPLATE: VALUE OF THE CARDS



Each card has its own value from 1 to 7

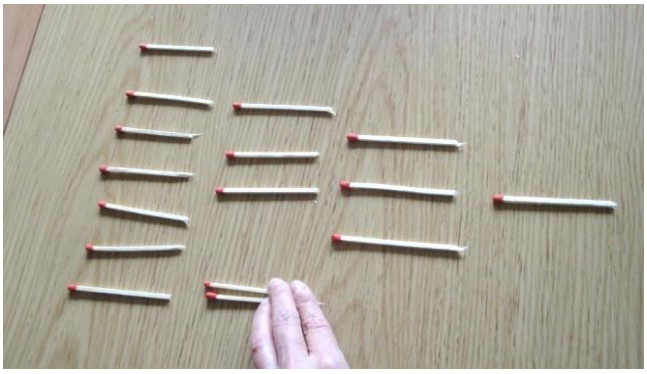
Figures value is 0.5 points

Players fill in this table after each 3 games

	Example	Game 1	Game 2	Game 3
Total number of cards	**4 cards**			
Value of each card	**3 + 0,5 + 1 + 2**			
Total value	**6,5**			

9.3 NIM GAME (GAME WITH MATCHES)

OBJECTIVES



- Count reliably up to 10 items.
- Add single-digit numbers with totals to 10.
- Subtract single-digit numbers from numbers up to 10.
- Understand probability in order to decide the number of matches to remove.
- Identify the possibilities of success each time we remove a match.

TOOLS, MATERIALS AND ORGANISATION

- Sit the students in pairs opposite each other
- Take 16 matches (or other objects like pebbles, pencils...) for each 2 players.
- Prepare copies from the initial pattern for each student.
- The lesson takes 45 minutes.

DESCRIPTION OF THE LESSON

First part of the lesson:

- Give the players the 16 matches and a copy of the initial pattern.
- Tell the players to put the 16 matches on the table following the initial pattern.
- Practice taking out a given number of matches from only one row and counting how many matches remain.

Second part of the lesson:

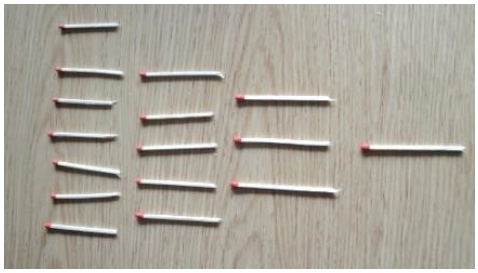
- Explain how to play the Nim game.
- The participants play several times the Nim game until they have learnt how the game works.
- Swap the players every five hands in order to practice the strategy against another player.
- Put the students all together in a big group.
- Talk about the strategy they have used to try to win the game.

USEFUL HINTS

- Be sure the participants have understood the operation of the game.
- Change the opponents often in order to test different strategies of playing.
- The teacher should make students think over the strategy they can use to win the game.

WORKSHEET 9.3 (LEARNER)

BEFORE PLAYING



How many matches are there in each row?

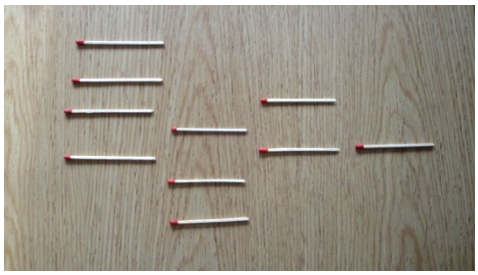
First _____

Second _____

Third _____

Fourth _____

DURING THE GAME



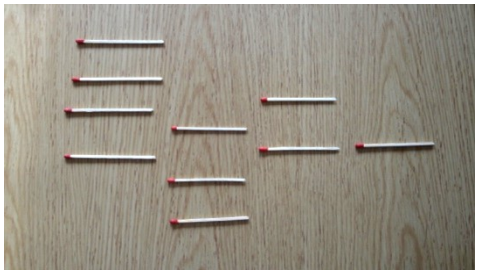
How many matches have been removed from each row?

First _____

Second _____

Third _____

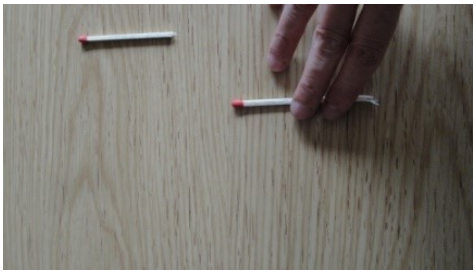
Fourth _____



How many matches have been removed in total? _____

How many matches are left on the board?

_____



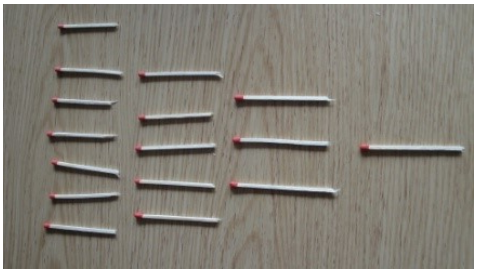
Who started the game? _____

Who removed the last match? _____

Does a player take advantage of being the first in playing? _____

WORKSHEET 9.3 (TEACHER)

BEFORE PLAYING



How many matches are there in each row?

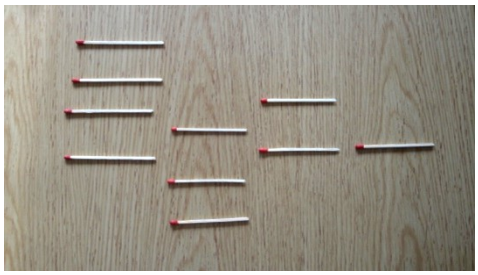
First *There are 7 matches*

Second *There are 5 matches*

Third *There are 3 matches*

Fourth *There is only 1 match*

DURING THE GAME



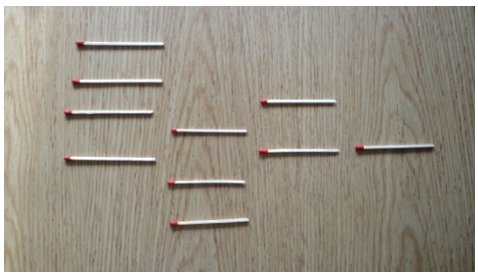
How many matches have been removed from each row?

First *7 – 4= 3 matches have been taken*

Second *5 – 3= 2 matches have been taken*

Third *3 – 2= 1 match has been taken*

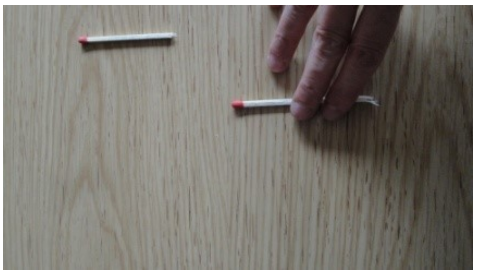
Fourth *1 – 1= 0 no match has been taken*



How many matches have been removed in total? *3 + 2 + 1 + 0 = 6 matches in total*

How many matches are left on the board?

10 matches are left on the board

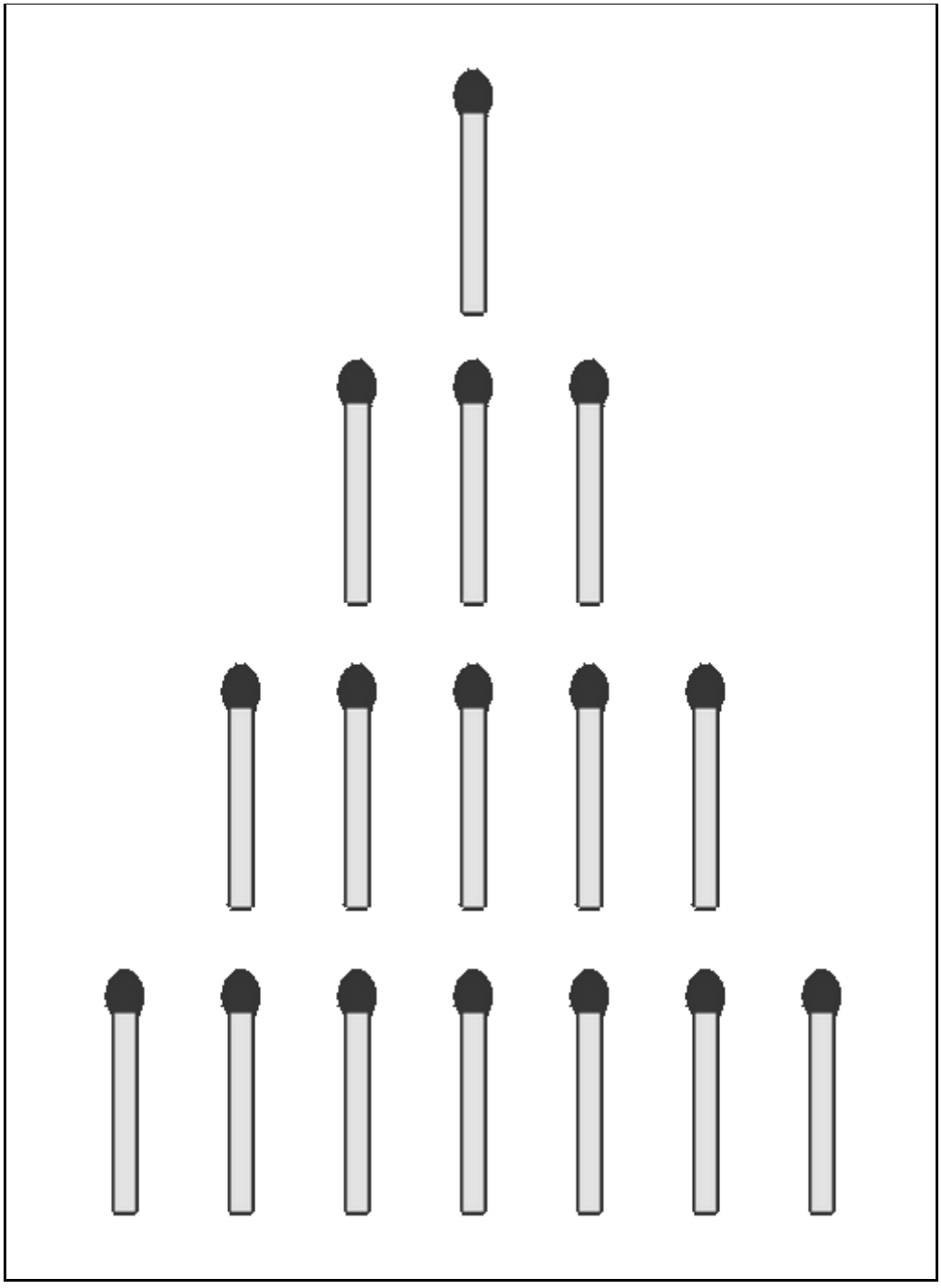


Who started the game? _____

Who removed the last match? _____

Does a player take advantage of being the first in playing? _____

Create a discussion about the player's strategy to win the game.

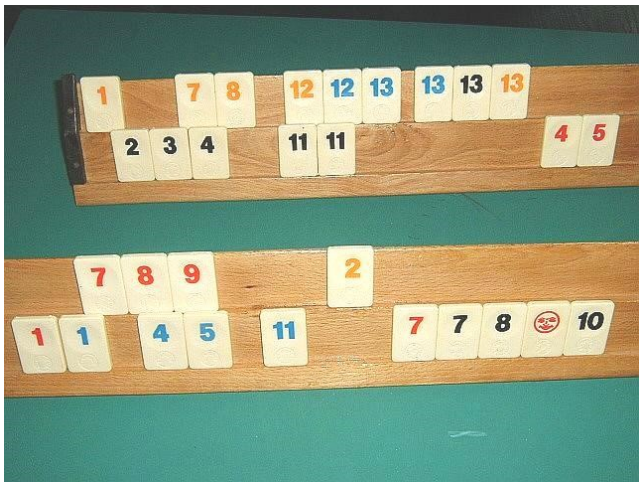


	Games				
Players	1	2	3	4	5
A					
B					

Fill in this table after playing against an opponent marking with an **X** for the winner of each game!

10.1 OKAY - RUMMIKUB (BOARD GAME)

OBJECTIVES



- Participants already can count from 1 to 13.
- You can differentiate colours.
- The game Rummikub is similar to the card game Rummy.
- You learn to combine 13 numbers from 1 to 13 in 4 different colours.
- One learns to observe and apply mathematical and game rules.

TOOLS, MATERIALS AND ORGANISATION

- Always 4 players play in a group together.
- Each player gets 14 tiles, the player, who will begin, gets 15 tiles.
- You can buy Rummikub in an shop or you construct your tiles on your own (see the copying template).

DESCRIPTION OF THE LESSON

First part of the lesson (10 minutes)

- Explain the game Rummikub.
- Use for this explanation the work sheet for students.
- Each single student gets his own work sheet.
- Edit the worksheet together.

Second part of the lesson (30 minutes)

- Sit together in groups with 4 participants.
- Each participant gets 14 tiles, the player, who will begin, get 15 tiles.
- Play according to the rules. A whole game can sometimes last longer than 30 minutes.

USEFUL HINTS

Rummikub in the Internet:

- http://rummikub.spiel-jetzt.org/
- https://en.wikipedia.org/wiki/Rummikub
- https://www.youtube.com/watch?v=yLsXoE3si7E
- https://www.youtube.com/watch?v=yrOYpkxL3G0

WORKSHEET 10.1 (LEARNER)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Calculation:

1. Basic knowledge:

Add the missing numbers!

Note:

Each digit can only appear once in the field of the number line.

2. How many tiles are in a Rummikub game?

- There are stones from 1 to 13
- Each stone is available twice
- Each stone is available in 4 colours
- Two joker stones are added

3. In what combinations can the stones be put together?

- Condition 1:
  3 or 4 stones with same number but in different colour
- Condition 2:
  Sequences composed of at least 3 consecutive stones of the same colour.
- Condition 3:
  You can lay out all the stones if you have exactly 7 pairs of the same value and the same colour.

Condition 1:

red	black	blue	

Condition 2:

4 red				
12 black				

Condition 3 (very rare):

6	6	.	.	.	.	.	.
10	10	.	.	.	.		

WORKSHEET 10.1 (TEACHER)

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Calculation:

13 stones

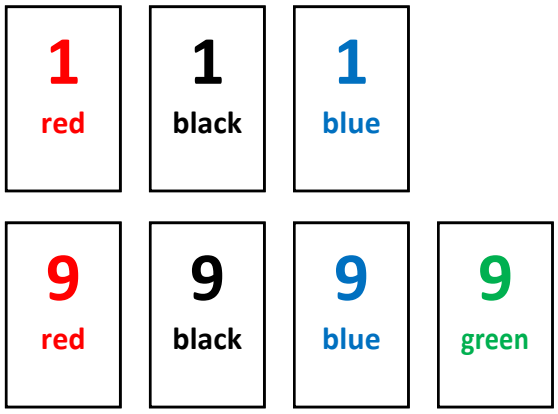
13 stones + 13 stones

* 4 colours

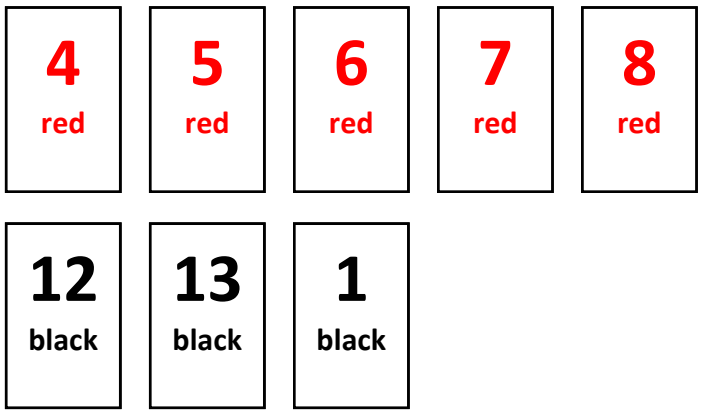
+ 2 jokers

= 106 stones

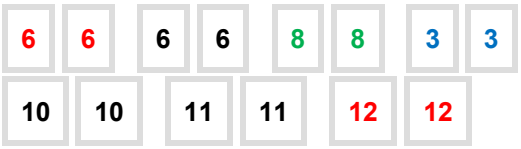
Condition 1:



Condition 2:



Condition 3 (very rare):



1. Basic knowledge:

Add the missing numbers!

Note:

Each digit can only appear once in the field of the number line.

2. How many tiles are in a Rummikub game?

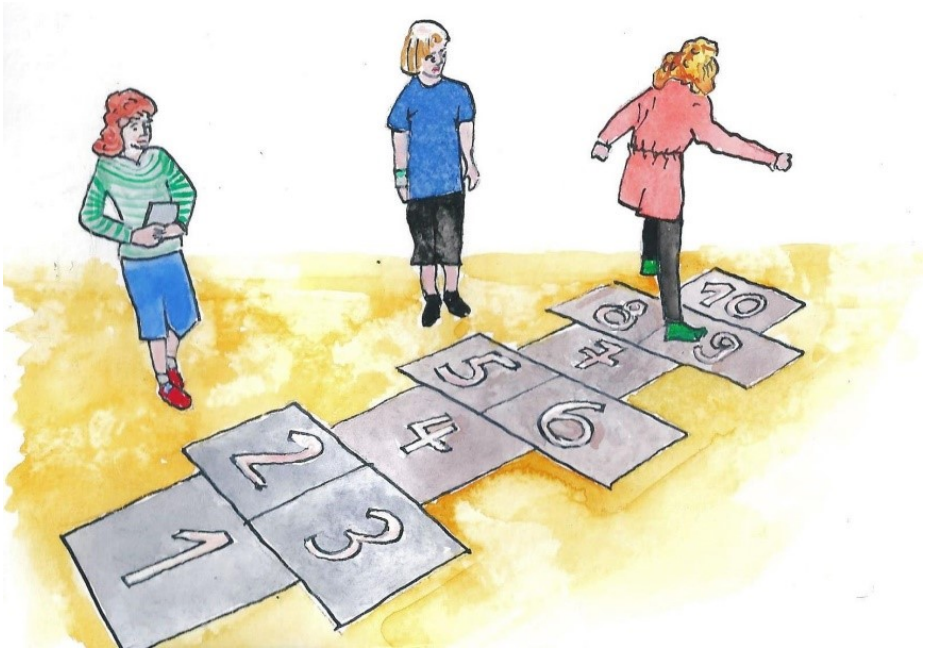
- There are stones from 1 to 13
- Each stone is available twice
- Each stone is available in 4 colours
- Two joker stones are added

3. In what combinations can the stones be put together?

- Condition 1:
  3 or 4 stones with same number but in different colour
- Condition 2:
  Sequences composed of at least 3 consecutive stones of the same colour.
- Condition 3:
  You can lay out all the stones if you have exactly 7 pairs of the same value and the same colour.

10.2 Hopscotch (Outside Game)

Objectives



- Count reliably up to 10 items, add and subtract single digit numbers
- Drawing two dimensional figures (rectangle and circle)
- By playing HOPSCOTCH the participants will surely learn the numbers from 1 to 10
- The learning content is also better absorbed by physical activity during hopping

Tools, Materials and Organisation

- Groups of 3 to 5 players play together.
- A paved place and chalk for drawing

Description of the Lesson

First part of the lesson (20 minutes)

- Explain the game HOPSCOTCH.
- Use the work sheet for students for this explanation.
- Each single student gets his own work sheet.
- Edit the worksheet together.

Second part of the lesson (20 minutes)

- Go out on the pavement in groups with 3 to 5 participants.
- Each group gets chalk for drawing on the pavement.
- Play according to the rules - have fun and learn the number row.

Useful Hints

Various variations of the rules make the game diverse.

Hopscotch in the Internet:

- https://en.wikipedia.org/wiki/Hopscotch

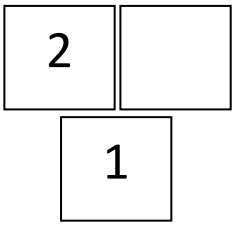
WORKSHEET 10.2 (LEARNER)

0 1 5 6 10

1. Basic knowledge:

Add the missing numbers!

Note:



2. See this photo and draw on the left side your own hopscotch-figure.

3. See this photo and draw on the left side your own hopscotch-figure with four branches.

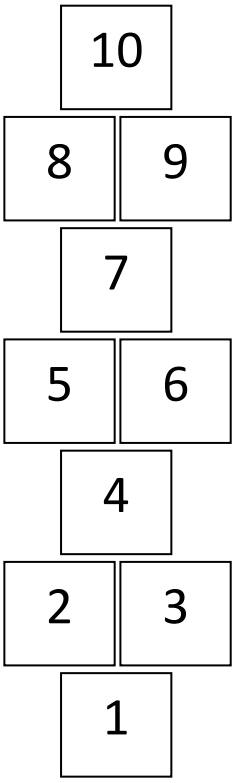
4. Go out to the pavement and draw with chalk your own hopscotch-figure!

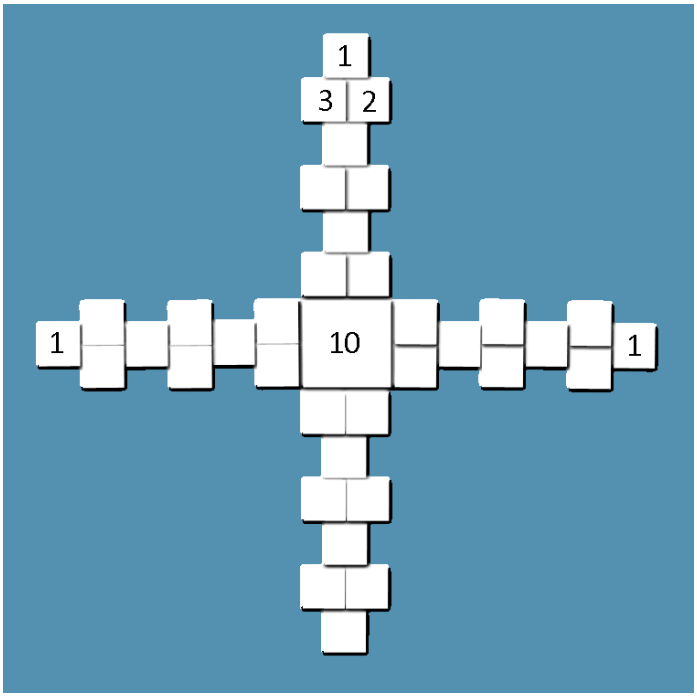
Agree the rules!

Play and have fun.

WORKSHEET 10.2 (TEACHER)

0 1 2 3 4 5 6 7 8 9 10





1. Basic knowledge:

Add the missing numbers!

Note:

Each digit can only appear once in the field of the number line.

2. See this photo and draw on the left side your own hopscotch-figure.

3. See this photo and draw on the left side your own hopscotch-figure with four branches.

4. Go out to the pavement and draw with chalk your own hopscotch-figure!

Agree the rules!

Play and have fun.

10.3 SUDOKU (PAPER-PENCIL GAME)

		7	3	6			9	2
					8			1
1	8	2					7	
				3	4			
	2		9	8	7		5	
			1	2				
	6					4	8	9
8			6					
2	1			9	5	3		

OBJECTIVES

- Participants already can count from 1 to 9.
- They know that there are 9 single digit numbers from 1 to 9
- Order, write and compare numbers from 1 to 9
- You learn to combine 9 numbers between 1 and 9, you find missing numbers or you eliminate double numbers.
- One learns to observe and apply mathematical rules.

TOOLS, MATERIALS AND ORGANISATION

- Each player gets one Sudoku board (3 by 3 small fields combined to 81 fields).
- The Sudoku playing field must be prepared, because not every number combination is possible. You can use different websites to produce Sudoku playing fields (see useful hints)
- The Sudoku game is a solitary game, you play it alone.
- The lesson takes 45 minutes. First you explain the rules, then the participants can play one or more games. Please start with easy Sudoku playing fields.

DESCRIPTION OF THE LESSON

First part of the lesson (5 minutes):

- Explain the game Sudoku.
- Use the work sheet for the students for this explanation.
- Each single student gets his own work sheet.
- Edit the worksheet together.

Second part of the lesson (40 minutes):

- Each participant gets his own work sheet.
- Each participant fills his Sudoku field.
- When it is finished, the teacher checks the solution.
- In case of difficulties the teacher may help the participant.
- It is important that all participants find the right combination of numbers from 1 to 9.

USEFUL HINTS

The game SUDOKU is related to the rules a very easy game, but it can be very complicated during playing.

Sudoku Generator in the Internet:

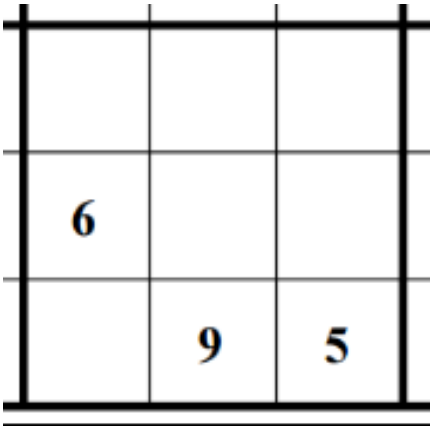
- https://www.kompf.de/sudoku/generator.html
- http://www.websudoku.com/
- http://www.opensky.ca/~jdhildeb/software/sudokugen/

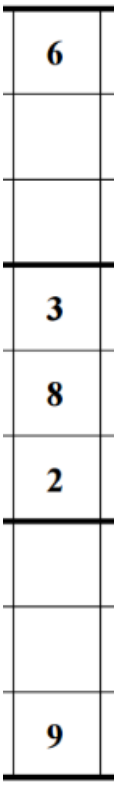
WORKSHEET 10.3 (LEARNER)

0 1 10 11

1. Basic knowledge:

Add the numbers by the missing digits!

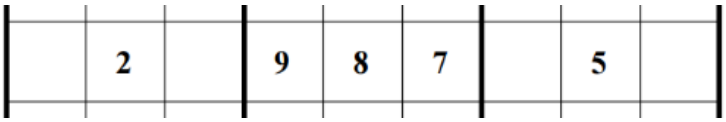




Note:

Each digit can only appear in the field.

2. Add the fields by numbers and observe the rule: Any number between 1 and 9 may only occur once in the 9 fields!

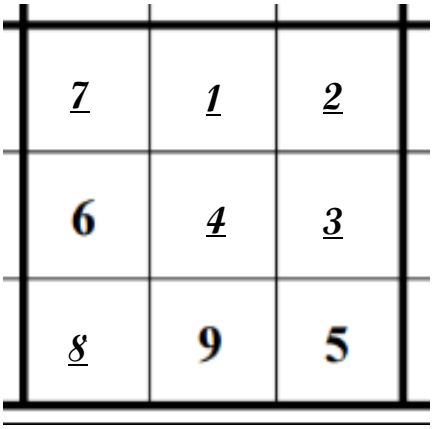


3. Working together the first simple example of a Sudoku field!

		7	3	6			9	2
					8			1
1	8	2					7	
				3	4			
	2		9	8	7		5	
			1	2				
	6					4	8	9
8			6					
2	1			9	5	3		

WORKSHEET 10.3 (TEACHER)

0 1 2 3 4 5 6 7 8 9 10 11



6
7
5
3
8
2
1
4
9

4	2	3	9	8	7	1	5	6

1. Basic knowledge:

Add the numbers by the missing digits!

Note:

Each digit can only appear once in the field.

2. Add the fields by numbers and observe the rule: Any number between 1 and 9 may only occur once!

Any number between 1 and 9 can be used as long as it occurs only once.

5	4	7	3	6	1	8	9	2
9	3	6	2	7	8	5	4	1
1	8	2	4	5	9	6	7	3
6	9	1	5	3	4	7	2	8
4	2	3	9	8	7	1	5	6
7	5	8	1	2	6	9	3	4
3	6	5	7	1	2	4	8	9
8	7	9	6	4	3	2	1	5
2	1	4	8	9	5	3	6	7

3. Working together the first simple example of a Sudoku field!

On the left side you see the solution!

MATHEMATICAL GLOSSARY[2]

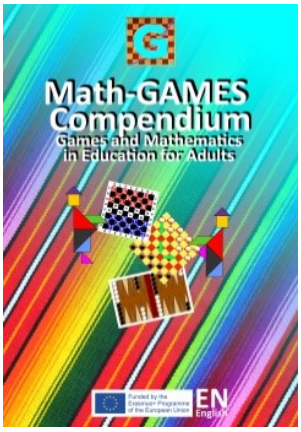
analogue clock	an analogue clock usually has 12 equal divisions around the perimeter/ circumference, labelled 1 to 12 to represent hours. It has two hands that rotate about the centre. The hour hand completes one revolution in 12 hours, and the minute hand completes one revolution in one hour.
angle	a configuration of two line segments meeting at a point. The term is often used for the measure of rotation from one of the line segments to the other. In this sense, a right angle measures 90°, an acute angle is less than 90°, an obtuse angle is greater than 90° but less than 180°, and a reflex angle is greater than 180°.
approximation	a result that is not exact but sufficiently close to be useful in a practical context. Verb: approximate. Adverb: approximately.
area	a measure of a surface. Measured in squares, e.g. square centimetres (cm^2), square metres (m^2).
average	sometimes used synonymously with arithmetic 'mean'. More widely, measures of average include mean, median and mode.
bar chart	a particular form of representation of data. Bars of equal width represent frequencies where the lengths are proportional to the frequencies. The bars may be presented vertically or horizontally.
block graph	a particular form of representation of grouped data. In its simplest form, where the class intervals are equal and rectangles have bases of the same size, the block graph can be considered as a bar chart, and the length of each rectangle represents the total in each class.
calculate efficiently	use knowledge of number systems and operations, e.g. use multiplication rather than repeated addition. In the context of using tools, to use available operations and functions, e.g. memory and constant functions on a calculator, sum formula in a spread sheet for a range of cells, rather than addition of individual cells.
capacity	volume, i.e. a measure in three-dimensional space, applied to liquids, materials that can be poured, or containers. Units include cubic centimetres (cm^3), cubic metres (m^3). Note – a litre is 1000 cm^3 (the volume of 1 kg of water).
cardinal number	a counting number. Example: one, two, three, etc.
combined events	a set of independent events with a single outcome. An independent event does not influence a subsequent event: for example, one throw of a die does not influence a second throw. Two throws of a die is a combined event with 36 possible outcomes (6 x 6). The probability of throwing two sixes is 1/36.
common	an adjective used to describe units, instruments, measures, date formats, etc. that are widely used in everyday life in non-specialist contexts.
common fraction	a fraction where the numerator and denominator are both integers. This is also known as a simple or vulgar fraction.
commutative	an operation * is commutative if a * b = b * a. Addition and multiplication are commutative where 2 + 3 = 3 + 2 and 2 * 3 = 3 * 2. Subtraction and division are not commutative.
continuous data	data resulting from measurement, e.g. length, temperature. Continuous data can take any value between two values, and can only be measured approximately to a certain degree of accuracy. A line usually represents continuous data.

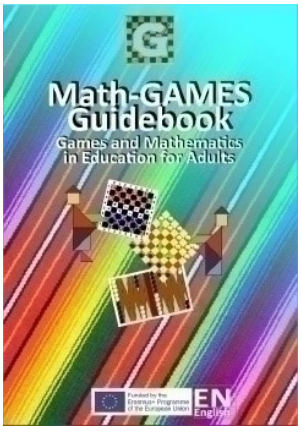
[2] adapted and modified from the Adult Core Curriculum, The Basic Skills Agency 2001, Produced by Cambridge Training and Development Ltd. on behalf of the Basic Skills Agency, Commonwealth House, 1–19 New Oxford Street, London WC1A 1NU.

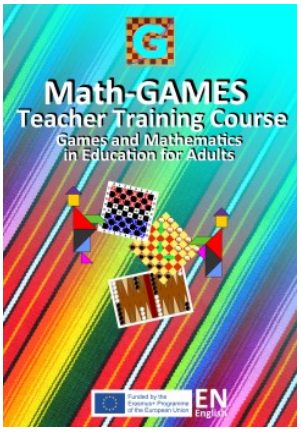
coordinates	clearly state a point in a coordinate system.
coordinate system	can be clearly determined e. g. by two number lines.
cuboid	a three-dimensional figure with six rectangular faces. Adjoining edges and faces are at right angles. (One pair of opposite faces may be square.)
data	information of a quantitative nature consisting of counts or measurements: where they refer to items or events that are separate and can be counted, the data are discrete; where they refer to quantities such as length or capacity that are measured, the data are continuous. Singular: datum.
decimal	relating to base ten. This is used most commonly synonymously with decimal fraction, where the number of tenths, hundredths, thousandths, etc. is represented as digits following a decimal point. The decimal point is placed at the right of the units' column. Each column after the decimal point is a decimal place: for example, the decimal fraction 0.275 is said to have three decimal places. The system of recording with a decimal point is decimal notation.
digit	one of the symbols of a number system, most commonly the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The number 29 is a two-digit number, for example, but there are three digits in 2.95. The position of the digits conveys place value.
digital clock	a digital clock is usually a 24-hour clock. It displays the time as hours and minutes past midnight: for example, four-thirty in the afternoon is displayed as 16:30. A 12-hour digital clock displays hours past midnight and midday and uses a.m. and p.m. to differentiate.
direct proportion	two quantities or variables are in direct proportion when they increase or decrease in the same ratio: for example, if 5 oranges cost £1, and 10 cost £2, then cost is in direct proportion to quantity – they both double, or both halve; expressed mathematically as y = k * x where k is constant.
discrete data	data resulting from a count of separate items or events, e.g. number of people.
equivalent fraction	fraction with the same value as another, e.g. ½ = 3/6 = 6/12. These fractions are equivalent.
estimate	to arrive at a rough answer by calculating with suitable approximations for numbers or, in measurement, by using previous experience.
evaluate	to work out the value of an expression when numbers have been substituted for variables.
event	used in probability to describe the outcome of an action or happening.
everyday	an adjective used to describe numbers, measures, units, instruments, etc. that fall within the daily lived experience of most people in non-specialist contexts.
expression	a mathematical statement involving variables written in words or symbols, e.g. length x width, a x b (or a·b).
factor	when an integer can be expressed as the product or two or more other integers, these are factors of the first. Example: 24 = 6 x 4, so 6 and 4 are factors of 24. A prime factor is a factor that is a prime number.
formula	any identity, general rule or mathematical law. Plural: formulae.
integer	any of the positive or negative whole numbers including zero, e.g. …-2, -1, 0, 1, 2 …
inverse operations	operations that, when they are combined, leave the entity on which they operate unchanged: for example, addition and subtraction are inverse operations, e.g. 5 + 6 - 6 = 5; multiplication and division are inverse operations, e.g. 6 * 10 / 10 = 6.
line	or straight line is the shortest connection between two points and on the points beyond.
line segment	is the shortest connection between two points, which begins and ends at these points.

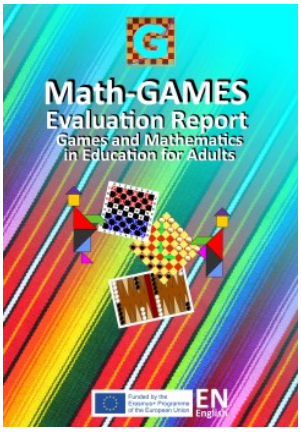
line symmetry	also reflective symmetry. The property of a shape where one half is a reflection of the other; the 'mirror line' is the axis of symmetry or line of symmetry.
mass	a fundamental characteristic of a body, relating to the amount of matter within it. Mass differs from weight, the force with which a body is attracted towards the earth's centre. Under certain conditions a body can become weightless, whereas mass is constant.
mean	a type of average. The arithmetic mean is the sum of quantities divided by the number of them: for example, the arithmetic mean of 5, 6, 14, 15 and 45 is (5 + 6 + 14 + 15 + 45) / 5 = 17.
median	a type of average. The median is the middle number or value when all are arranged in ascending order: for example, the median of 5, 6, 14, 15 and 45 is 14. Where there is an even number of values, the arithmetic mean of the two middle values is calculated: for example, the median of 5, 6, 7, 8, 14 and 45 is (7 + 8) / 2 = 7.5.
mixed fraction	a whole number and a fractional part expressed as a common fraction, e.g. 1 2/3 is a mixed fraction; also known as a mixed number.
multiple	any number that has a given number as a factor is called a multiple of that factor. Example: 14 = 7 * 2, 49 = 7 * 7 and 70 = 7 * 10. So 14, 49 and 70 are all multiples of 7.
negative number	a number less than 0.
number line	a line with sorted numbers on it
operation	a means of combining numbers, sets or other elements. Addition, subtraction, multiplication and division are four operations on numbers.
ordinal number	a term that describes a position within an order, e.g. first, second, third, fourth . . . twentieth, etc.
parallel	always equidistant. Parallel lines never meet, however far they are produced. Parallel lines must lie in the same plane.
percentage	a fraction expressed as the number of parts per hundred and recorded using the notation number%, e.g. one half can be expressed as 50%, the whole can be expressed as 100%.
perimeter	the length of the boundary of a closed figure.
pi (π)	the symbol used to denote the ratio of the circumference of a circle to its diameter. Approximately 3.142
pie-chart	a particular form of presentation of data. A circle is divided into sectors. The frequency or amount of each quantity is proportional to the angle at the centre of the circle.
place value	the value of a digit that relates to its position or place in a number, e.g. in 1481 the digits represent thousands, hundreds, tens and units respectively. The value of the 1 on the left is one thousand while the value of the 1 on the right is one.
prime number	a prime number has exactly two factors, itself and 1. Examples: 2 (factors 2, 1), 3 (factors 3, 1); 51 is not prime (factors 51, 17, 3, 1), and 1 itself is not considered a prime number.
probability	the likelihood of an event happening; a measure of certainty. Probability is expressed on a scale from 0 to 1. Where an event cannot happen, its probability is 0 and, where it is certain, its probability is 1. The probability of scoring 1 with a fair dice is 1/6, because the dice has 6 sides.
product	the result of multiplication, e.g. the product of 2, 3 and 4 is 24 (2 * 3 * 4).
property	any attribute, e.g. one property of a square is that all sides are equal.
range	a measure of spread in statistics; the difference between the greatest and the least in a set of numerical data.

ratio	a comparison of quantities of the same kind, written a:b; for example, a mixture made up of two ingredients in the ratio 3:1 is 3 parts of the first ingredient to 1 part of the second; the first ingredient makes up ¾ of the total mixture, the second makes up ¼ of the total.
rectangle	a quadrilateral (four-sided polygon) with four right angles. The pairs of opposite sides are equal. If all sides are equal the rectangle is a square. Adjective: rectangular.
regular	a polygon is a regular polygon if all the sides are equal and all the internal angles are equal, e.g. a regular quadrilateral is a square. When referring to a shape, the adjective regular refers to common 2-D or 3-D shapes whose areas can be found using a formula, e.g. a rectangle, circle, cylinder.
right angle	one quarter of a complete turn. It is an angle of 90 degrees. An acute angle is less than one right angle. An obtuse angle is greater than one right angle but less than two. A reflex angle is greater than two right angles.
round (verb)	to express a number or measurement to a required degree of accuracy, e.g. 543 rounded to the nearest 10 is 540.
scale	a measuring device usually consisting of points on a line with equal intervals.
sequence	a succession of terms formed according to a rule, in which there is a definite relation between one term and the next and between each term and its position in the sequence, e.g. 1, 4, 9, 16, 25, etc.
sign	a symbol used to denote an operation, e.g. addition sign +, subtraction sign -, multiplication sign x, division sign /. In the case of directed numbers, the positive + or negative - sign indicates the direction in which the number is measured from the origin along the number line.
simple	an adjective applied to numbers, information, diagrams, charts, etc. that make limited demands on the learner, for example: small whole numbers; numbers that are easier to work with, e.g. multiples of 2, 5 10, 100; uncomplicated representations of limited amounts of data, etc.
square number	a number that can be expressed as the product of two equal numbers, e.g. 36 = 6 x 6, and so 36 is a square number.
standard unit	units that are agreed throughout a community, e.g. the metre is a standard unit of length. Non-standard units are not widely agreed.
straight line	is the shortest connection between two points
symbol	a letter, numeral or other mark that represents a number, an operation or another mathematical idea: for example, V is the Roman symbol for five, > is the symbol for 'is greater than'.
symmetry	a figure has symmetry if parts can be interchanged without changing the whole. A geometric figure may have reflective symmetry or rotational symmetry. Adjective: symmetrical.
unit fraction	a fraction that has 1 as the numerator and whose denominator is a non-zero integer, e.g. ½, 1/3, …
unique line	part of a line, which has a beginning and a end.
volume	a measure in three-dimensional space, measured in cubes, e.g. cubic centimetres (cm^3), cubic metres (m^3).
weight	the force with which a body is attracted towards the earth's centre. In non-scientific contexts it is often used synonymously with mass (though technically different). Metric units of weight include kilograms (kg) and grams (g).









AVAILABLE MATERIAL IN THE MATH-GAMES PROJECT

The four parts of the output of the project Math-GAMES:

- "**Math-GAMES Compendium of Famous Traditional Games**", which are books in ten languages **(BG, DE, EN, ES, VA, FR, GR, IT, RO, TR)**. After that the partners of the project will prove how traditional games could be implemented in their learning program for a better understanding Mathematics, especially for lower skilled people, for young people and for immigrants, if there are needs.
- The results are the "**Math-GAMES Numeracy Learning Guidebook (Mathematical Literacy)**" in nine languages **(BG, DE, EN, ES, VA, FR, GR, IT, RO)**.
- In the third part of the project the project partners will prove by doing and testing during real courses and seminars that playing games between people with different skills assist in social integration and thus traditional games will be saved by transferring them to other people and they will not be lost. The result is a "**Math-GAMES Teacher Training Course and Seminar**", which is held for the next years in different countries. The e-presentation, the seminar and the teacher-training course are published in English.
- Finally the "**Math-GAMES Testing and Evaluation Report**" will be published. It is a report about the project, the work, the activities during the lessons, the competitions in schools, the meetings and the evaluation. The Math-GAMES Project Report is published in English. All the Material is available from 2018.

Information:
Website: www.math-games.eu
E-Mail: roland.schneidt@web.de

Link to the special Math-GAMES YouTube Channel:
https://www.youtube.com/channel/UCvuYRVDPN WRNO5SwQiRre4g